inkMaths

ANSWER BOOK

Circles

Coordinate geometry · Year 12–13

7 questions · 22 marks

NAVY EDITION

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SECTION A · Warm-up

A1 $(x - 2)^2 + (y + 3)^2 = 16$. M1 for the form with the centre in place, A1 for the square of the radius. The signs flip on the way in, and the radius is squared. Both of the usual slips live in that one line. [2]

A2 Centre $(-1, 5)$, radius $\sqrt{49} = 7$. B1 for the centre, B1 for the radius. Reading $(x + 1)$ as centre +1 is the flip in reverse. Quoting the radius as 49 is the other frequent miss, and it costs the second mark on its own. [2]

SECTION B · Standard

B1 Complete the square in each variable. $(x - 2)^2 - 4 + (y + 3)^2 - 9 - 3 = 0$, so $(x - 2)^2 + (y + 3)^2 = 16$, giving centre $(2, -3)$ and radius 4. M1 for completing the square in both variables, A1 for the centre, A1 for the radius. All three loose constants must migrate to the right before you take the square root; leaving the -3 behind is worth a mark. The same form settles where a point sits: substitute it into the left side and compare with 16, so $(5, 1)$ gives $9 + 16 = 25$ and lies outside. [3]

B2 Check the point first. $3^2 + 4^2 = 25$, so it does lie on the circle. The radius from $(1, 2)$ to $(4, 6)$ has gradient $4/3$, so the tangent's gradient is the negative reciprocal, $-3/4$. Through $(4, 6)$: $y - 6 = -(3/4)(x - 4)$, which clears to $3x + 4y - 36 = 0$. M1 for the gradient of the radius, M1 for the negative reciprocal, A1 for the equation through the point, A1 for the demanded form. The perpendicular gradient and the demanded form are two separate pieces of work, so an answer left as $y = \ldots$ has finished only one of them. No calculus appears anywhere; the perpendicular-radius fact does all the work. [4]

B3 The perpendicular from the centre bisects the chord, so it makes a right triangle with hypotenuse 5 and one leg 3. The half-chord is $\sqrt{(25 - 9)} = 4$, and the chord is 8 cm. M1 for the right triangle with hypotenuse 5 and one leg 3, A1 for the chord. Stopping at 4 is the slip the word half exists to prevent. [2]

SECTION C · Stretch

C1 Substitute the line into the circle so that only x survives: $x^2 + (2x - 3)^2 - 6x + 4(2x - 3) - 12 = 0$. Expanding gives $5x^2 - 10x - 15 = 0$, and dividing by 5 leaves $x^2 - 2x - 3 = 0$, so $(x - 3)(x + 1) = 0$ and $x = 3$ or $x = -1$. Feeding each back into the line, A(3, 3) and B(-1, -5). M1 for substituting the line into the circle, A1 for the reduced quadratic, M1 for solving it, A1 for the x values, A1 for both points. Substitution must go into the circle, never the other way round, and the y values come from the line because the arithmetic there is trivial. Two real roots means the line is a secant; a repeated root would have meant a tangent, and no real root a clean miss. [5]

C2 $3^2 + 4^2 = 25$, so C is on the circle. The vector from C to A is (-8, -4) and from C to B is (2, -4), giving gradients 1/2 and -2, whose product is -1. So CA is perpendicular to CB. B1 for the substitution showing C is on the circle, M1 for the two gradients, A1 for their values, A1 for the product and the conclusion. This is the angle-in-a-semicircle theorem caught in the act, and any other point of the circle would behave the same way. A show-that question wants the substitution written out; the theorem's name is no substitute for the working. [4]