inkMaths

ANSWER BOOK

Combinations of normal random variables

Further Statistics 2 · Year FM

6 questions · 24 marks

NAVY EDITION

inkphysics.co.uk/maths/ · youtube.com/@InkPhysics

SECTION A · Warm-up

A1 It is normal, with mean $a\mu_x + b\mu_y$ and variance $a^2\sigma_x^2 + b^2\sigma_y^2$. The variables must be independent; without that the variances cannot simply be added. B1 for the mean and variance, B1 for the independence condition. [2]

A2 X + Y is **N(19,** 25) and X - Y is **N(5,** 25). The means follow the sign, but the variances add in both cases, so both have standard deviation 5. [4]
For 2X - 3Y the mean is 2(12) - 3(7) = 24 - 21 = 3, and the variance is $2^2(9) + 3^2(16) = 36 + 144 = 180$. So 2X - 3Y is **N(3,** 180), with standard deviation 13.4.
B1 for the sum, B1 for the difference, M1 for the mean and variance of the combination, A1 for its distribution.
Every coefficient squares, including the one attached to a subtraction. Writing 2(9) - 3(16) is the usual mistake, and it produces a negative variance.

SECTION B · Standard

B1 $P(X < Y) = P(X - Y < 0)$, and X - Y is N(5, 25) with standard deviation 5. [4]
Standardising: $z = (0 - 5)/5 = -1$, so the probability is $P(Z < -1)$ = **0.1587**. M1 for turning the comparison into a single variable, A1 for its distribution, M1 for standardising, A1 for 0.1587. Using variance 9 - 16 would give a negative variance, which is the check that the rule must be addition.

B2 Five independent readings: mean 60, variance 5 × 9 = 45, so **N(60,** 45) with standard deviation 6.71. Five times one reading: mean 60, variance $5^2 \times 9 = 225$, so **N(60,** 225) with standard deviation 15. M1 A1 for the distribution of the sum, A1 for the distribution of the multiple, B1 for the explanation. The five separate readings vary independently and their errors partly cancel; scaling one reading magnifies its error fivefold. [4]

B3 The total for eight people is normal with mean 8(70) = 560 and variance 8(100) = 800, so the standard deviation is $\sqrt{800} = 28.28$. The variance is multiplied by 8, not by 64; that is the distinction between eight people and one person weighed eight times. [4]
Standardising: $z = (600 - 560)/28.28 = 1.414$, so P(total > 600) = 1 - 0.9214 = **0.0786**, about one time in thirteen.
M1 A1 for the mean and variance of the total, M1 for standardising, A1 for 0.0786.
The independence assumption is doing real work here. A family travelling together is unlikely to satisfy it.

SECTION C · Stretch

C1 Let n be the number of adults. The total weight is $N(70n, 100n)$, with standard deviation $10\sqrt{n}$. Note the root; using $10n$ instead is the error that makes this question look easier than it is. [6]

The requirement $P(\text{total} > 600) < 0.01$ means the value 600 must lie at least 2.3263 standard deviations above the mean, since $\Phi(2.3263) = 0.99$. So $(600 - 70n)/(10\sqrt{n}) > 2.3263$.

Substituting $t = \sqrt{n}$ turns this into a quadratic: $600 - 70t^2 > 23.263t$, that is $70t^2 + 23.263t - 600 < 0$. Making that substitution is the method mark; there is no way through by inspection.

The positive root is $t = 2.766$, so $n < 7.65$ and the greatest whole number is **n** = 7.

M1 for the distribution of the total in terms of n, B1 for the value 2.3263, M1 for the inequality, M1 for the substitution $t = \sqrt{n}$, A1 for the positive root, A1 for the greatest whole number.

Check both sides. With $n = 7$ the z value is 4.16 and the probability is 0.000016, comfortably below 0.01. With $n = 8$ the z value is 1.41 and the probability is 0.0786, far above it. Rounding 7.65 up to 8 would break the condition, so the direction of the rounding carries a mark of its own.