inkMaths

ANSWER BOOK

Combinatorics

Further Pure 2 · Year FM

6 questions · 20 marks

NAVY EDITION

SECTION A · Warm-up

A1 Ordered: $10 \times 9 \times 8 \times 7 =$ **5040**, which is ${}^{10}P_4$. Unordered: divide by the $4! = 24$ orderings of each selection, giving **210** $= {}^{10}C_4$. M1 A1 for the ordered count, A1 for the unordered count. The two always differ by exactly $r!$. [3]

A2 Fix one person to remove the rotational freedom, then arrange the other 7 in the remaining seats: $7! =$ **5040** ways. M1 for fixing one person, A1 for 5040. Counting $8!$ would treat each arrangement as 8 different ones, once for each rotation. [2]

SECTION B · Standard

B1 There are 10 letters, with S appearing 3 times, T 3 times and I twice; A and C appear once each. Dividing the $10!$ orderings by the indistinguishable rearrangements gives $10!/(3!\ 3!\ 2!) = 3628800/72 =$ **50400**. M1 for $10!$ over the repeats, A1 for the divisor, A1 for the total. [3]

B2 Total committees: ${}^{12}C_5 = 792$. Count the complement instead. No women: ${}^{7}C_5 = 21$. Exactly one woman: $5 \times {}^{7}C_4 = 5 \times 35 = 175$. So the answer is $792 - 21 - 175 =$ **596**. M1 for a complete method, A1 for the no-women count, A1 for the one-woman count, A1 for 596. Adding the cases with 2, 3, 4 and 5 women gives the same total in four times the work. [4]

B3 Multiples of 3: 166. Multiples of 5: 100. Multiples of 15, counted in both: 33. By inclusion and exclusion the answer is $166 + 100 - 33 =$ **233**. M1 for the three counts, A1 for subtracting the overlap, A1 for 233. Forgetting the overlap gives 266 and counts every multiple of 15 twice. [3]

SECTION C · Stretch

C1 The total number of subsets is $2^{10} = 1024$, since each element is independently in or out. Expanding $(1 - 1)^{10}$ by the binomial theorem gives the alternating sum of the ${}^{10}C_r$, which is 0, so the even-sized and odd-sized subsets are equal in number. Each group therefore has $1024/2 =$ **512** members. B1 for the total number of subsets, M1 for the binomial expansion, A1 for the alternating sum being zero, M1 for concluding that the two groups are equal, A1 for the printed result. [5]