inkMaths

ANSWER BOOK

Complex arithmetic and the Argand diagram

Complex numbers · Year FM

7 questions · 23 marks

NAVY EDITION

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SECTION A · Warm-up

A1 $z + w = 4 - 2i$ and $z - w = 2 + 6i$. B1 for the sum, B1 for the difference. Real parts combine with real, imaginary with imaginary; nothing crosses over. [2]

A2 $zw = 3 - 12i + 2i - 8i^2 = 3 - 10i + 8 = 11 - 10i$. M1 for the expansion, A1 for the answer. The whole calculation is expanding brackets, with $i^2 = -1$ the only new step, converting the $-8i^2$ into $+8$. [2]

SECTION B · Standard

B1 Multiply top and bottom by the conjugate $1 + i$. The denominator becomes $(1 - i)(1 + i) = 2$ and the numerator $(2 + 3i)(1 + i) = 2 + 2i + 3i - 3 = -1 + 5i$. So the quotient is $-1/2 + (5/2)i$. M1 for multiplying by the conjugate, A1 for the real denominator, A1 for the quotient. Realising the denominator is the entire method. [3]

B2 $|z| = \sqrt{(9 + 16)} = 5$. z sits at the point $(3, 4)$ in the first quadrant; the conjugate $3 - 4i$ sits at $(3, -4)$. Conjugation is reflection in the real axis, and both points are distance 5 from the origin. B1 for the modulus, B1 for the two positions, B1 for the reflection. [3]

B3 The discriminant is $16 - 52 = -36$, so $z = (4 \pm 6i)/2 = 2 \pm 3i$. The roots are a conjugate pair, as the roots of any quadratic with real coefficients must be when they are not real. M1 for the discriminant, A1 for both roots, B1 for naming them a conjugate pair. The $\pm$ of the quadratic formula lands on $\pm 6i$. [3]

B4 Expanding gives $(2x + y) + (2y - x)i = 5$. Matching real and imaginary parts gives $2x + y = 5$ and $2y - x = 0$, so $x = 2y$ and $5y = 5$, giving $y = 1$ and $x = 2$. Then $x + yi = 2 + i$, and $(2 + i)(2 - i) = 4 + 1 = 5$ as a check. M1 for the expansion, A1 for equating real and imaginary parts, A1 for y, A1 for x. One complex equation always carries two real ones. [4]

SECTION C · Stretch

C1 Real coefficients force the conjugate $2 - i$ to be a root as well, so $(z - 2 - i)(z - 2 + i) = z^2 - 4z + 5$ is a factor. Write the cubic as $(z^2 - 4z + 5)(z - r)$. The constant term is $-5r$, and matching it to 10 gives $r = -2$, so the third root is -2. Expanding, $z^3 - (4 + r)z^2 + (5 + 4r)z - 5r$, so $a =$ **-2** and $b =$ **-3**. Checking the sum of the roots, $(2 + i) + (2 - i) + (-2) = 2 = -a$. B1 for the conjugate root, M1 A1 for the quadratic factor, M1 for the remaining linear factor, A1 for the third root, A1 for a and b. Reaching for the conjugate first is what keeps this to real arithmetic; substituting $z = 2 + i$ and expanding cubes wastes the structure. [6]