inkMaths

ANSWER BOOK

Compound angles and the harmonic form

Trigonometry · Year 12–13

7 questions · 30 marks

NAVY EDITION

SECTION A · Warm-up

A1 Write $75°$ as $45° + 30°$ and expand. $\cos 75° = \cos 45° \cos 30° - \sin 45° \sin 30° = (\sqrt{2}/2)(\sqrt{3}/2) - (\sqrt{2}/2)(½) = (\sqrt{6} - \sqrt{2})/4$. M1 for the expansion with the correct middle sign, A1 for the simplified surd. Cosine flips the sign in the middle, so a plus inside the bracket becomes a minus outside. [2]

A2 From the 3-4-5 triangle, $\cos A = 4/5$, positive because A is acute. Then $\sin 2A = 2 \sin A \cos A = 2 \times (3/5)(4/5) = 24/25$. B1 for cos A, M1 for quoting the double angle formula, A1 for 24/25. Doubling the angle is not doubling the sine. An answer of 6/5 gives that mistake away instantly, since no sine can exceed 1. [3]

SECTION B · Standard

B1 The cosine-only form settles the first part in one step. $\cos 2A = 2 \cos^2 A - 1 = 2/9 - 1 = -7/9$. For sin 2A you do need sin A, and $\sin A = \sqrt{(1 - 1/9)} = \sqrt{(8/9)} = 2\sqrt{2}/3$, taking the positive root because A is acute. Then $\sin 2A = 2 \sin A \cos A = 2 \times (2\sqrt{2}/3)(1/3) = 4\sqrt{2}/9$. M1 A1 for cos 2A, B1 for sin A, M1 A1 for sin 2A. Notice that cos 2A comes out negative while A itself is acute. Doubling has carried the angle past $90°$, and students who expect the sign to follow A talk themselves out of a correct answer. [5]

B2 Expanding $R \sin (\theta + \alpha) = R \cos \alpha \sin \theta + R \sin \alpha \cos \theta$ and matching coefficients gives $R \cos \alpha = 5$ and $R \sin \alpha = 12$. Square and add for $R = \sqrt{(25 + 144)} = 13$; divide for $\tan \alpha = 12/5$, so $\alpha = 1.176$. Hence $5 \sin \theta + 12 \cos \theta = 13 \sin (\theta + 1.176)$. The maximum is 13, reached when the sine equals 1, so $\theta + 1.176 = \pi/2$ and $\theta = 0.395$. The marks run M1 for the method for R, A1 for 13, M1 for tan α, A1 for 1.176, B1 for the maximum, A1 for θ. Writing $\tan \alpha = 5/12$ is the standard slip, and it costs the last three marks in one go. [6]

B3 Expand the double angle to get $2 \sin x \cos x = \sin x$, gather everything on one side and factorise, giving $\sin x (2 \cos x - 1) = 0$. From $\sin x = 0$ come $x = 0°$ and $180°$; from $\cos x = ½$ come $x = 60°$ and $300°$. All four are needed: $0°, 60°, 180°, 300°$. M1 for the double angle expansion, M1 for the factorisation, A1 A1 for the four solutions. Dividing both sides by sin x at the start looks tidy and silently destroys two of them, and that is exactly why the factorisation is the step worth showing. [4]

SECTION C · Stretch

C1 Fold the left-hand side first. Expanding $R\cos(\theta + \alpha) = R\cos\alpha\cos\theta - R\sin\alpha\sin\theta$ and matching gives $R\cos\alpha = 3$ and $R\sin\alpha = 4$, so $R = 5$ and $\tan\alpha = 4/3$, that is $\alpha = 0.9273$. The equation becomes $5\cos(\theta + 0.9273) = 2$, so $\cos(\theta + 0.9273) = 0.4$. Now $\theta + 0.9273$ runs over $[0.9273, 0.9273 + 2\pi)$, and cosine takes the value 0.4 there at 1.1593 and at $2\pi - 1.1593 = 5.1239$. Subtracting α gives $\theta = 0.232$ and $\theta = 4.197$. M1 for the expansion and matching, A1 for R, M1 A1 for α, A1 A1 for the two solutions. The trap is stopping at the principal value the calculator returns. Work out the full interval for $\theta + \alpha$, list every solution inside it, and subtract α only at the very end. [6]

C2 Expand both compound angles: $(\sin A\cos B + \cos A\sin B) + (\sin A\cos B - \cos A\sin B)$. The $\cos A\sin B$ terms cancel and the two $\sin A\cos B$ terms add, leaving $2\sin A\cos B$ as required. For the second part, 75° and 15° average 45° and differ by 30°, so take $A = 45°$ and $B = 30°$. Then $\sin 75° + \sin 15° = 2\sin 45°\cos 30° = 2 \times (\sqrt{2}/2)(\sqrt{3}/2) = \sqrt{6}/2$. M1 for expanding both compound angles, A1 for the printed identity, M1 for choosing the two angles, A1 for the exact value. Half the sum with half the difference finds them every time. [4]