inkMaths

ANSWER BOOK

Definite integrals and areas

Integration · Year 12–13

6 questions · 22 marks

NAVY EDITION

SECTION A · Warm-up

A1 Antidifferentiate inside square brackets, $[x^4/4]$, then substitute. $16/4 - 0 = 4$. M1 for the antiderivative in brackets, A1 for 4. Upper limit first, lower limit subtracted. No + c appears, since the same constant would sit in both substitutions and cancel. [2]

A2 $[x^2 + x]$ from 1 to 4 = 20 - 2 = 18. M1 for the antiderivative, A1 for 18. Geometry checks it. The region is a trapezium with parallel sides 3 and 9 and width 3, and $\frac{1}{2}(3 + 9) \times 3 = 18$. Marks go missing here when the limits are substituted the wrong way round. [2]

SECTION B · Standard

B1 Find the roots before anything else. $6x - x^2 = x(6 - x) = 0$ at $x = 0$ and $x = 6$, and the arch lies above the axis between them. Area = $\int_0^6 (6x - x^2)\,dx = [3x^2 - x^3/3]$ from 0 to 6 = 108 - 72 = 36. M1 for solving for the roots, A1 for 0 and 6, M1 for the integration, A1 for 36. A candidate who invents limits rather than solving for them loses the first method mark. [4]

B2 The curve dips below the axis between its roots 0 and 2, then climbs above it. Split there. With $F(x) = x^3/3 - x^2$, the integral from 0 to 2 is $-4/3$, so that piece has area $4/3$, and from 2 to 3 it is $+4/3$. Total area $8/3$. B1 for splitting at the root, M1 for integrating a piece, A1 for $-4/3$, A1 for the second piece, A1 for the total $8/3$. A single integral from 0 to 3 returns 0, because the signed pieces cancel. The split and the change of sign are the whole of this question, so an unsplit answer of zero has not addressed it. [5]

B3 Divide through by $\sqrt{x}$ first, giving $x^{3/2} + 3x^{-1/2}$. Integrating term by term gives $(2/5)x^{5/2} + 6x^{1/2}$. At $x = 4$ that is $64/5 + 12 = 124/5$, and at $x = 1$ it is $2/5 + 6 = 32/5$. The difference is $92/5$, as required. M1 for dividing through by $\sqrt{x}$, A1 for the two powers, M1 for integrating, A1 for $92/5$. No rule integrates a quotient directly, so the algebra has to happen before the integral sign acts. A show-that question also wants the exact fraction, never 18.4. [4]

SECTION C · Stretch

C1 Solve $x^2 - 2x = 3$ for the intersections. $x^2 - 2x - 3 = (x - 3)(x + 1) = 0$, so C and l meet at $x = -1$ and $x = 3$. Between those values the line lies above the curve, so integrate line minus curve. $\int_{-1}^{3} (3 + 2x - x^2)\,dx = [3x + x^2 - x^3/3]$ from -1 to $3 = 9 - (-5/3) = 32/3$, as required. M1 for equating and solving, A1 for the limits -1 and 3, M1 for integrating line minus curve, A1 for the antiderivative, A1 for $32/3$. Two slips dominate. Subtracting the wrong way round returns $-32/3$, and integrating the curve alone throws away the strip of area between the curve and the line. A sketch, or a single test value such as $x = 0$, settles which graph is on top and secures the method mark. [5]