



QUESTION WORKBOOK

Diagonalisation and the Cayley–Hamilton theorem

Further Pure 2 · Year FM

6 questions · 23 marks

NAVY SCREEN EDITION

SECTION A · Warm-up

- A1** The matrix M has rows $(4, 1)$ and $(2, 3)$, with eigenvalues 5 and 2 and corresponding eigenvectors $(1, 1)$ and $(1, -2)$. Write down matrices P and D such that $M = PDP^{-1}$.

[2]

- A2** State the Cayley-Hamilton theorem, and verify it for the matrix M with rows $(4, 1)$ and $(2, 3)$.

[3]

SECTION B · Standard

- B1** The matrix M with rows $(4, 1)$ and $(2, 3)$ has eigenvalues 5 and 2 with eigenvectors $(1, 1)$ and $(1, -2)$. Use a diagonalisation to find M^5 .

[5]

- B2** The matrix M has rows $(4, 1)$ and $(2, 3)$. Use the Cayley-Hamilton theorem to express M^{-1} in terms of M and I , and hence evaluate M^{-1} . [4]

- B3** Give a 2 by 2 matrix that cannot be diagonalised, and explain what goes wrong. [3]

SECTION C · Stretch

- C1** The matrix M with rows $(4, 1)$ and $(2, 3)$ has eigenvalues 5 and 2. Show that for every positive integer n , $M^n = a_n M + b_n I$ where $a_n = (5^n - 2^n)/3$, and find b_n . Hence evaluate M^6 . [6]