inkMaths

ANSWER BOOK

Differentiating trig, exponentials and logs

Differentiation · Year 12–13

9 questions · 29 marks

NAVY EDITION

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SECTION A · Warm-up

A1 sin 2x gives 2 cos 2x, and cos 3x gives -3 sin 3x. B1 B1, one for each derivative. [2]
The inner coefficient multiplies out to the front, and cosine's derivative carries the minus sign.

A2 e^{5x} gives $5e^{5x}$, which is k times itself. ln x gives 1/x. B1 B1, one for each derivative. [2]
The logarithm is the one entry on the shelf whose derivative leaves its own family entirely, and that is worth memorising rather than deriving under exam pressure.

SECTION B · Standard

B1 Work termwise. 2 sin 3x gives 6 cos 3x, and 4 cos 2x gives -8 sin 2x, so dy/dx = 6 cos 3x - 8 sin 2x. M1 for a correct attempt at either term, then A1 A1. Each term's inner coefficient multiplies its own front constant, so the 3 pairs with the 2 and the 2 pairs with the 4, never the other way about. [3]

B2 dy/dx = 5/x + 2 cos 2x. At x = π, cos 2π = 1, so the gradient is 5/π + 2 = 3.59. M1 A1 for the derivative, M1 for the substitution, A1 for the value. The logarithm's derivative still has an x in it, so the gradient genuinely depends on where you measure it. Working in degrees turns cos 2π into cos 6.28° and quietly ruins the answer. [4]

B3 The chord gradient is [sin (x + h) - sin x]/h. Expanding the compound angle gives [sin x cos h + cos x sin h - sin x]/h, and regrouping gives sin x (cos h - 1)/h + cos x (sin h)/h. Letting h → 0 and quoting the two given limits leaves 0 + cos x × 1 = cos x. M1 for the chord gradient, M1 for the compound-angle expansion, M1 for the regrouping, A1 for the conclusion. The proof needs radians throughout, since (sin h)/h → 1 is false in degrees. [4]

B4 dy/dx = $2e^{2x} + 4e^{-x}$. The -1 in the second exponent meets the -4 in front and the term turns positive, which is the sign most often lost. At x = 0 both exponentials equal 1, so the gradient is 2 + 4 = 6. M1 A1 for the derivative, A1 for the value. [3]

B5 tan kx differentiates to k $\sec^2$ kx, so dy/dx = 3 $\sec^2$ 3x. At x = 0, sec 0 = 1 and the gradient is 3. M1 A1 A1. Since $\sec^2$ is never less than 1, this gradient never drops below 3, so tan 3x has no stationary points anywhere. [3]

SECTION C · Stretch

C1 From $x = e^y$, differentiate with respect to y to get $dx/dy = e^y$. Inverting, $dy/dx = 1/e^y$, and since $e^y = x$ this is $1/x$ as required. M1 for dx/dy, M1 for turning the rate upside down, A1 for replacing e^y by x. The final substitution is the mark students forget, leaving the answer in terms of y when the question asked for x. [3]

C2 Write $a^x = e^{x \ln a}$, using $a = e^{\ln a}$. The exponent is now x times the constant ln a, so the chain rule gives $\ln a \times e^{x \ln a} = a^x \ln a$. For $y = 2^x$, $dy/dx = 2^x \ln 2$, and at $x = 3$ the gradient is $8 \ln 2 = 5.55$. M1 for rewriting in base e, M1 for differentiating, A1 for the printed result, M1 for the substitution, A1 for 5.55. Treating a^x like x^n and writing $x\,a^{x-1}$ is the classic wrong turn. The variable is upstairs, so the power rule has no claim on it. [5]