inkMaths

ANSWER BOOK

Direct impact and Newton's law of restitution

Further Mechanics 1 · Year FM

6 questions · 23 marks

NAVY EDITION

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SECTION A · Warm-up

A1 The speed of separation equals e times the speed of approach, both measured along the line of centres, with $0 \le e \le 1$. At $e = 1$ the impact is perfectly elastic and no kinetic energy is lost; at $e = 0$ the spheres do not separate at all and move off together. B1 for the law, B1 for the range with both ends explained. [2]

A2 Momentum: $3(6) = 3v_1 + 2v_2$, so $3v_1 + 2v_2 = 18$. Restitution: $v_2 - v_1 = 0.5(6) = 3$. Substituting: $3v_1 + 2(v_1 + 3) = 18$ gives $5v_1 = 12$, so $v_1 =$ **2.4** m/s and $v_2 =$ **5.4** m/s. M1 for conservation of momentum, M1 for the restitution equation, A1 for both velocities. [3]

SECTION B · Standard

B1 Before the impact: $\frac{1}{2}(3)(6^2) =$ **54** J, the stationary sphere contributing nothing. [4]
After: $\frac{1}{2}(3)(2.4^2) + \frac{1}{2}(2)(5.4^2) = 8.64 + 29.16 =$ **37.8** J. Both spheres must appear in this line; omitting the struck one is the usual slip and it inflates the loss.
The loss is $54 - 37.8 =$ **16.2** J, which is 30% of the original energy. M1 for the kinetic energy before, A1 for 54 J, M1 for the kinetic energy after with both spheres, A1 for a loss of 16.2 J.

B2 During the impact each sphere exerts a force on the other that is equal in size and opposite in direction, for the same length of time, so the impulses cancel and, provided any external impulse is negligible over the brief contact, the total momentum is unchanged. Kinetic energy obeys no such law. Some of it goes into deforming the spheres, and into sound and heat, and does not come back unless the impact is perfectly elastic. B1 for the equal and opposite forces acting for the same time, B1 for the impulses cancelling, B1 for the energy lost to deformation, sound and heat. [3]

B3 Fix a positive direction and keep it for every velocity. Taking the 4 kg sphere's original direction as positive gives $u_1 = 5$, $u_2 = -2$ and $v_1 = -1$. Sign errors here are the most common slip in the whole topic. [4]
Momentum: $4(5) + 6(-2) = 8$, so $4(-1) + 6v_2 = 8$ and $v_2 =$ **2** m/s.
Speed of approach $= 5 - (-2) = 7$, and speed of separation $= 2 - (-1) = 3$.
So $e = 3/7 =$ **0.429**, which lies between 0 and 1 as it must. M1 for conservation of momentum, A1 for the second velocity, M1 for the speeds of approach and separation, A1 for $e = 0.429$. A value outside that range means a sign has slipped.

SECTION C · Stretch

C1 Take the direction of u as positive, and let the velocities after the impact be v_A and v_B. [7]
Momentum: $3mu = 3mv_A + mv_B$, so $3v_A + v_B = 3u$.
Restitution: $v_B - v_A = eu$.
Substituting the second into the first gives $4v_A = 3u - eu$, so $\mathbf{v_A} = u(3 - e)/4$ and $\mathbf{v_B} = 3u(1 + e)/4$. Keep the answers as single fractions; the algebra that follows is much worse otherwise.
Kinetic energy before = $\frac{1}{2}(3m)u^2 = 3mu^2/2$.
After = $\frac{1}{2}(3m)v_A^2 + \frac{1}{2}(m)v_B^2 = (3mu^2/32)[(3 - e)^2 + 3(1 + e)^2]$.
Expanding the bracket: $(9 - 6e + e^2) + (3 + 6e + 3e^2) = 12 + 4e^2$. The terms in e cancel, which is the sign that the expansion is right. So the energy after is $3mu^2(12 + 4e^2)/32 = mu^2(9 + 3e^2)/8$.
Loss = $12mu^2/8 - mu^2(9 + 3e^2)/8 = \mathbf{3mu^2(1} - e^2)/8$, as required. M1 for the momentum equation, M1 for the restitution equation, A1 for both velocities, M1 for the kinetic energy before and after, A1 for the energy after, M1 for the subtraction, A1 for the printed result.
Two checks. At $e = 1$ the loss is zero, which is what perfect elasticity means. At $e = 0.5$ with $m = 1$ and $u = 6$, the formula gives 10.125 J, and the velocities 3.75 and 6.75 m/s give the same figure directly.