inkMaths

ANSWER BOOK

Discrete random variables and expectation

Further Statistics 1 · Year FM

6 questions · 22 marks

NAVY EDITION

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SECTION A · Warm-up

A1 The probabilities must total 1: $k(1 + 2 + 3 + 4) = 10k = 1$, so $k = 1/10$. M1 for summing the probabilities to 1, A1 for $k = 1/10$. Checking validity before anything else is what makes every later answer trustworthy. [2]

A2 $E(X) = 1(1/6) + 2(1/3) + 3(1/3) + 4(1/6) = 1/6 + 2/3 + 1 + 2/3 = 5/2$. M1 for $\Sigma x\, P(X = x)$, A1 for the four products, A1 for $5/2$. The distribution is symmetric about 2.5, so the mean sits exactly there, and no listing was strictly needed. [3]

SECTION B · Standard

B1 $E(X^2) = 1(1/6) + 4(1/3) + 9(1/3) + 16(1/6) = 1/6 + 4/3 + 3 + 8/3 =$ **$43/6$**. [4]
$E(X) = 5/2$ by symmetry about 2.5, so $\mathrm{Var}(X) = E(X^2) - [E(X)]^2 = 43/6 - 25/4 =$ **$11/12$**, about 0.917. M1 for $\Sigma x^2\, P(X = x)$, A1 for $43/6$, M1 for $E(X^2) - [E(X)]^2$, A1 for $11/12$.
The square goes on the mean alone. Writing $E(X^2 - X)^2$ or subtracting $5/2$ rather than its square is the slip that loses the accuracy mark here.

B2 $E(3X - 1) = 3(5/2) - 1 =$ **$13/2$**. $\mathrm{Var}(3X - 1) = 3^2 \times 11/12 =$ **$33/4$**. B1 for $13/2$, M1 for multiplying the variance by 3^2, A1 for $33/4$. [3]
The -1 shifts the centre and leaves the spread untouched, while the 3 enters the variance squared. Subtracting 1 from the variance as well is the standard error, and it is worth saying in your working that adding a constant cannot change a spread.

B3 $E(X) = 0(0.7) + 1(0.2) + 5(0.1) = 0.7$, so the expected payout is 70p. A stake above 70p loses money on average; a fair game charges exactly the expected payout. M1 for $\Sigma x\, P(X = x)$, A1 for an expected payout of 70p, B1 for a fair stake equalling the expected payout, B1 for 70p. Note that 70p is not a possible outcome, which is normal for a mean. [4]

SECTION C · Stretch

C1 Two unknowns need two equations, and the first is always that the probabilities total 1. [6]
$2a + 2b = 1$.
For the second, $E(X) = a(1) + a(2) + b(3) + b(4) = 3a + 7b = 2.7$. Collecting the coefficients of a and of b before substituting keeps the algebra short.
From the first equation $a = 0.5 - b$. Substituting: $3(0.5 - b) + 7b = 2.7$, so $1.5 + 4b = 2.7$ and $\mathbf{b} = 0.3$, giving $\mathbf{a} = 0.2$.
Check: the probabilities 0.2, 0.2, 0.3, 0.3 total 1, and $E(X) = 0.2 + 0.4 + 0.9 + 1.2 = 2.7$.
Now $E(X^2) = 0.2(1) + 0.2(4) + 0.3(9) + 0.3(16) = 0.2 + 0.8 + 2.7 + 4.8 = 8.5$, so $\mathrm{Var}(X) = 8.5 - 2.7^2 = 8.5 - 7.29 = \mathbf{1.21}$.
Finally $\mathrm{Var}(5 - 2X) = (-2)^2 \times 1.21 = \mathbf{4.84}$. B1 for $2a + 2b = 1$, M1 for the expectation equation, A1 for $a = 0.2$ and $b = 0.3$, M1 for $E(X^2)$, A1 for $\mathrm{Var}(X) = 1.21$, A1 for 4.84. The coefficient squares, so its sign disappears; a negative variance means the squaring was forgotten.