



QUESTION WORKBOOK

Eigenvalues and eigenvectors

Further Pure 2 · Year FM

6 questions · 27 marks

PRINT EDITION

SECTION A · Warm-up

- A1** M has rows $(4, 1)$ and $(2, 3)$. Find its eigenvalues and a corresponding eigenvector for each. [5]

- A2** Prove that if λ is an eigenvalue of M with eigenvector v , then λ^2 is an eigenvalue of M^2 with the same eigenvector. [3]

SECTION B · Standard

- B1** A has rows $(2, 0, 1)$, $(0, 3, 0)$ and $(1, 0, 2)$. Find its eigenvalues and an eigenvector for each distinct one. [7]

- B2** State how the trace and determinant of a 2 by 2 matrix relate to its eigenvalues, and verify both for the matrix in question 1. [3]

- B3** Explain what the eigenvectors of a 2 by 2 matrix mean geometrically for the transformation it represents. [3]

SECTION C · Stretch

- C1** S has rows $(3, 1)$ and $(1, 3)$. Find its eigenvalues and eigenvectors, and explain why a symmetric matrix with distinct eigenvalues always has perpendicular ones. [6]