inkMaths

ANSWER BOOK

Flows in networks: cuts and capacity

Decision Mathematics 2 · Year FM

6 questions · 22 marks

NAVY EDITION

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SECTION A · Warm-up

A1 9 + 6 = **15**. The arc crossing back towards the source contributes nothing. It carries flow away from the sink, so it can only reduce what gets through, never add to the ceiling the cut imposes. Adding all three capacities to get 20 is the standard error, and it produces a bound that is not even valid. B1 for the capacity, B1 for the treatment of the backward arc. [2]

A2 A cut separates the source from the sink, so every unit of flow arriving at the sink must cross the cut in the forward direction at least once. The forward arcs cannot carry more than their total capacity, so the flow cannot exceed that total. B1 for every unit crossing the cut, B1 for the capacity limit. [2]

SECTION B · Standard

B1 Round the source, the arcs are SA and SB: 12 + 9 = **21**. [5]
Round the sink, the arcs are CT and DT: 10 + 11 = **21**.
For the third cut the forward arcs are AC and DT: 7 + 11 = **18**, with no arc crossing back.
So the flow cannot exceed **18**, and neither of the two obvious cuts was the binding one.
B1 B1 for the first two cuts, M1 for the forward arcs of the third, A1 for 18, A1 for the bound on the flow.

B2 The arcs running forward across the cut are SA and BD, giving 12 + 8 = **20**, with no arc crossing back. [3]
Every cut is an upper bound, so the flow is at most 20 and also at most 18. The smaller bound is the only one that bites. M1 for the forward arcs SA and BD, A1 for 20, B1 for the smaller bound being the binding one. Quoting the first cut you happen to draw, rather than hunting for the smallest, is what costs the final accuracy mark.

B3 Add a **supersource** joined to each factory by an arc whose capacity is that factory's output, and a **supersink** reached from each shop by an arc whose capacity is that shop's demand. The network then has a single source and a single sink, and every method for cuts and flows applies without modification. B1 for the supersource and its capacities, B1 for the supersink and its capacities, B1 for the reduction to standard form. [3]

SECTION C · Stretch

C1 Split D into **D-in** and **D-out**. The arcs AD and BD now arrive at D-in, the arc DT now leaves D-out, and a single arc from D-in to D-out carries capacity **8**. Every route through D is forced along that arc, which is what the restriction means. Draw the split vertex; the method mark is for the diagram, not for a sentence about it. [7]

The cut now crosses AC and the new arc rather than DT, so its capacity is 7 + 8 = **15**, and the flow is at most 15.

Now exhibit a flow of 15: SA 7, SB 8, AC 7, AD 0, BD 8, CT 7, DT 8. Each arc is within capacity, D carries exactly 8, and flow in equals flow out at every intermediate vertex, so 15 units reach T.

A flow of 15 and a cut of 15 pin the answer between the same two numbers, so the maximum flow is **15**. A bound alone never finishes the argument; the marks for proving maximality are for the matching flow. M1 A1 for splitting D with an arc of capacity 8, M1 A1 for the cut capacity, M1 A1 for a feasible flow of that value, A1 for the conclusion.