



ANSWER BOOK

Functions in modelling

Algebra and functions · Year 12–13

7 questions · 24 marks

NAVY EDITION

SECTION A · Warm-up

- A1** Equal steps added: linear. Equal factors: exponential. Repeating on a fixed cycle: trigonometric. B1 B1 B1 for the three families. Diagnosis comes before fitting; the behavioural signature picks the family, and only then do constants get estimated. [3]
- A2** The sine runs between ± 1 , so h runs from $6 - 1.8 = 4.2$ m up to $6 + 1.8 = 7.8$ m. B1 B1 for the two depths. Centre line 6, swing 1.8: the two numbers in the model are exactly these two facts. [2]

SECTION B · Standard

- B1** The sine completes a cycle when $30t$ reaches 360, so the period is 12 hours. High tide needs $\sin(30t)^\circ = 1$, first at $30t = 90$, so $t = 3$ hours. B1 for the period, M1 for $30t = 90$, A1 for $t = 3$. Both answers come from reading the coefficient, not from any solving. [3]
- B2** At $t = 0$ the value is $400 + 5600 = \text{£}6000$. As t grows the exponential dies away, leaving the $\text{£}400$ floor: scrap value. At $t = 5$: $V = 400 + 5600e^{-1} = \text{£}2460$. B1 for $\text{£}6000$, B1 for $\text{£}400$, M1 for substituting $t = 5$, A1 for $\text{£}2460$. The two constants carry the two ends of the story: start, and long run. [4]
- B3** $400 + 5600e^{-0.2t} = 1200$ gives $e^{-0.2t} = 800/5600 = 1/7$. Taking logs: $0.2t = \ln 7$, so $t = 9.73$ years. M1 for isolating the exponential, A1 for $1/7$, M1 for taking logs, A1 for 9.73 years. The $\text{£}400$ must be moved across before the log is taken; logging a sum term by term is not a thing. [4]

SECTION C · Stretch

- C1** Predicting inside the fitted range is interpolation and is anchored on both sides. Everything past $t = 10$ is extrapolation, and its safety falls away with distance. At $t = 12$ the model has stepped only just outside its window and the growth conditions plausibly still hold, so the prediction is worth having. At $t = 100$ it claims around 2×10^{14} bacteria, and unbounded exponential growth needs unlimited food and space, which no dish supplies. The mathematics extends for ever. The conditions that justified it do not, and a model's authority stops where its conditions do. B1 for naming extrapolation, B1 for $t = 12$ sitting just outside the fitted range, B1 for the size of the prediction at $t = 100$, B1 for the biological limits. An answer that says only "the number is too big" gets one mark of the four; the marks want the fitted range named and the biology brought in. [4]

- C2** Set $6 + 1.8 \sin(30t)^\circ > 7$, so $\sin(30t)^\circ > 5/9 = 0.5556$. The principal angle is $30t = 33.75^\circ$, and sine stays above that value until its partner $180^\circ - 33.75^\circ = 146.25^\circ$. So $30t$ runs from 33.75° to 146.25° , giving t from 1.12 to 4.88 hours, a stretch of 3.75 hours in each 12-hour cycle. M1 for the inequality in sine, A1 for 33.75° , M1 for the second angle, A1 for 3.75 hours. The second angle is where the marks are lost: an inequality in sine needs both ends of the window, and a calculator hands over only the first. Sketching one arch of the curve and marking the line $h = 7$ across it settles the direction of the inequality in a moment.
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