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ANSWER BOOK

Functions, inverses and the modulus

Algebra and functions · Year 12–13

7 questions · 22 marks

NAVY EDITION

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SECTION A · Warm-up

A1 fg means g first, so $g(2) = 4$ and then $f(4) = 11$. The other order gives $f(2) = 5$ and then $g(5) = 25$. B1 B1 for the two values. The two answers differ, as composites usually do, and reading fg from the left is the commonest way to lose both marks at once. [2]

A2 $fg(x) = f(x^2) = 3x^2 - 1$, and $gf(x) = (3x - 1)^2$. B1 B1 for the two composites. The inner function's output is fed through the outer one whole, brackets and all. Writing $gf(x)$ as $3x^2 - 1$ as well is the slip that shows the bracket was dropped. [2]

SECTION B · Standard

B1 Write $y = x^2 - 3$ and solve for x, giving $x = \sqrt{(y + 3)}$. The positive root is the one to take, because the domain restricts x to be at least 0. Swapping letters, $f^{-1}(x) = \sqrt{(x + 3)}$. The range of f is $y \geq -3$, so the domain of f^{-1} is $x \geq -3$. Domain and range swap roles along with the letters, and the stated domain is worth a mark on its own. M1 for making x the subject, A1 for $\sqrt{(x + 3)}$, A1 for taking the positive root, B1 for the domain $x \geq -3$. [4]

B2 $y = (2x + 1)/5$ unwinds a step at a time. Multiply by 5 to get $5y = 2x + 1$, then $x = (5y - 1)/2$. Swapping letters, $f^{-1}(x) = (5x - 1)/2$. The inverse undoes the machine in reverse order, multiplying by 5 first where f divided by 5 last. M1 A1 for making x the subject, A1 for the inverse. [3]

B3 Take the two arms separately. Original arm: $3x - 2 = x + 4$ gives $x = 3$, and the check $|7| = 7$ holds. Reflected arm: $-(3x - 2) = x + 4$ gives $2 - 3x = x + 4$, so $x = -½$, and $|-3.5| = 3.5 = -½ + 4$ holds too. Both solutions survive, so $x = -½$ and $x = 3$. M1 A1 for the first arm, M1 A1 for the reflected arm. Every arm's answer has to be checked back, because reflected algebra can invent crossings the graphs never make. [4]

SECTION C · Stretch

C1 An inverse has to send each output back to a single input, but x^2 sends both 3 and -3 to 9. The function is not one-to-one, so no inverse exists. Restricting the domain to $x \geq 0$, or equally to $x \leq 0$, keeps one branch of the parabola and the inverse appears. B1 for the two inputs giving one output, B1 for the function not being one-to-one, B1 for a suitable restriction. Saying "because it is a curve" names no property; one-to-one is the property being asked about. [3]

C2 Two moduli means two cases, not four, because the overall sign of one side may be absorbed into the other. Same signs: $2x + 1 = x - 4$ gives $x = -5$. Opposite signs: $2x + 1 = -(x - 4) = 4 - x$ gives $3x = 3$, so $x = 1$. Checking, $x = -5$ gives $|-9| = |-9|$ and $x = 1$ gives $|3| = |-3|$, so both hold and $x = -5$ or $x = 1$. M1 A1 for the same-signs case, M1 A1 for the opposite-signs case. Squaring both sides is the clean alternative, since $(2x + 1)^2 = (x - 4)^2$ leads to $3x^2 + 12x - 15 = 0$ and the same pair; it also removes the need to check, because squaring cannot introduce a false root when both sides were already non-negative. [4]