



QUESTION WORKBOOK

Functions of two variables

Further Pure 2 · Year FM

15 questions · 64 marks

NAVY SCREEN EDITION

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SECTION A · Warm-up**A1**
FT-A1For $f(x, y) = x^2y^3 + 5x$, find f_x and f_y .

[3]

A2
FT-A2A surface has equation $z = x^2 + 3y^2$. Write down the section along $x = 2$, and describe the contour at $z = 12$.

[3]

A3
FT-A3For $f(x, y) = x^3 + 2x^2y - y^4$, find the four second partial derivatives, and verify that the mixed ones agree.

[4]

A4
FT-A4The surface S has equation $z = x^2 + y^2 - 4$. Write S in the implicit form $g(x, y, z) = c$, and state what the number z measures for a point of S lying above the point (x, y) of the horizontal plane.

[2]

SECTION B · Standard**B1**
FT-B1Find the stationary point of $f(x, y) = x^2 + y^2 - 4x + 6y + 3$ and determine its nature.

[4]

B2
FT-B2Find the four stationary points of $f(x, y) = x^3 - 3x + y^3 - 3y$ and classify each one.

[6]

B3
FT-B3Find the equation of the tangent plane to $z = x^2y - 3y^2$ at the point where $x = 2$ and $y = 1$.

[4]

B4
FT-B4

For $f(x, y) = e^{3x} \cos y$, find f_x , f_y and f_{xy} , and evaluate f_{xy} at $(0, \pi/2)$.

[4]

B5
FT-B5

For $f(x, y, z) = x^2y + yz^3 + 4z$, find f_x , f_y and f_z , and evaluate f_z at $(1, 2, -1)$.

[4]

B6
FT-B6

The surface S has equation $z = x^2 - y^2$. Describe the section $x = 1$ and the section $y = 0$, describe the contour $z = 0$, and say what shape the surface has near the origin.

[4]

B7 For $f(x, y) = \ln(x^2 + y^2)$, find f_x and f_y , and show that $f_{xx} + f_{yy} = 0$ wherever f is defined. [4]
FT-B7

SECTION C · Stretch

C1 Find the stationary points of $f(x, y) = x^4 + y^4 - 4xy$ and classify each one. [6]
FT-C1

C2 Show that the Hessian test gives no verdict at the origin for $f(x, y) = x^4 + y^4$, and determine the nature of that stationary point by another argument. [4]
FT-C2

C3

FT-C3

The temperature at the point (x, y) of a flat plate is modelled by $T = 100 - 2x^2 - y^2$ degrees. Find the tangent plane to this surface at the point where $x = 3$ and $y = 2$, use it to estimate the temperature at $(3.1, 1.9)$, and compare that estimate with the exact value. [6]

C4

FT-C4

A box with a rectangular base and no lid is to hold 32 cm^3 . Its base measures $x \text{ cm}$ by $y \text{ cm}$ and its height is $z \text{ cm}$. Show that the outside surface area is $S = xy + 64/x + 64/y$ square centimetres, and find the dimensions that make S least, justifying that the value is a minimum. [6]

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