



ANSWER BOOK

Further kinematics: acceleration as a function of x , t or v

Further Mechanics 2 · Year FM

6 questions · 22 marks

NAVY EDITION

SECTION A · Warm-up

- A1** Use $v \, dv/dx$. Writing dv/dt instead leaves an equation containing v , t and x , [2]
which cannot be separated because t and x are both unknown functions of each other.
Matching the form to the variable the acceleration depends on is what makes the
equation separable. B1 for choosing $v \, dv/dx$, B1 for why dv/dt fails.
- A2** The acceleration depends on t , so $dv/dt = 6t - 2$ and $v = 3t^2 - 2t + c$. With $v = 1$ [3]
at $t = 0$, $c = 1$, so $v = 3t^2 - 2t + 1$. At $t = 3$: $27 - 6 + 1 = \mathbf{22 \, m/s}$. M1 for integrating, A1 for the
expression for v , A1 for $22 \, m/s$.

SECTION B · Standard

- B1** The acceleration depends on v and distance is wanted, so use $v \, dv/dx =$ [4]
 $-0.02v^2$. Cancelling one v , which is valid while the particle is moving, gives $dv/v = -0.02$
 dx , so $\ln v = -0.02x + c$. With $v = 20$ at $x = 0$, $v = 20e^{-0.02x}$. At $x = 50$: $v = 20e^{-1} = \mathbf{7.36 \, m/s}$. M1
for using $v \, dv/dx$, M1 for separating and integrating, A1 for $v = 20e^{-0.02x}$, A1 for $7.36 \, m/s$.
- B2** Time is wanted and the acceleration depends on v , so $dv/dt = -0.4v$, giving $v =$ [4]
 $15e^{-0.4t}$. Setting $v = 7.5$: $e^{-0.4t} = 0.5$, so $0.4t = \ln 2$ and $t = \mathbf{1.73 \, s}$. M1 for $dv/dt = -0.4v$, A1 for
the expression for v , M1 for setting $v = 7.5$, A1 for $1.73 \, s$. The same time would halve it
again, so the speed never actually reaches zero.
- B3** At the terminal speed the acceleration is zero, so $2 = 0.05v^2$ and $v^2 = 40$, giving [3]
 $v = \mathbf{6.32 \, m/s}$. M1 for setting the acceleration to zero, A1 for $v^2 = 40$, A1 for $6.32 \, m/s$. No
integration is needed. The terminal speed is simply where the driving effect and the
resistance balance, and setting up an integral here wastes most of the time available.

SECTION C · Stretch

- C1** The acceleration depends on the displacement, so write it as $v \, dv/dx$. Using dv/dt here leaves three variables in one equation and goes nowhere, and choosing the right form is the first method mark. [6]
- Taking the direction of motion as positive, the acceleration is negative: $v \, dv/dx = -6/x^2$. Separating and integrating: $\int v \, dv = \int -6x^{-2} \, dx$, so $v^2/2 = 6/x + c$.
- At $x = 2$, $v = 2$, giving $2 = 3 + c$ and $c = -1$. Hence $v^2 = 12/x - 2$. Find the constant before going any further; carrying it symbolically to the end is where the arithmetic usually breaks.
- At $x = 4$: $v^2 = 3 - 2 = 1$, so the speed is **1** m/s.
- The particle is furthest from O when it is instantaneously at rest, so set $v = 0$: $12/x = 2$ and $x = 6$ m. Beyond that point v^2 would be negative, which is the confirmation that 6 m is the limit.
- M1 for $v \, dv/dx = -6/x^2$, M1 A1 for integrating, A1 for $c = -1$, A1 for the speed at $x = 4$, A1 for the greatest distance.
-