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ANSWER BOOK

Further loci and regions in the Argand diagram

Further Pure 2 · Year FM

6 questions · 25 marks

NAVY EDITION

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SECTION A · Warm-up

A1 Rewrite the first as $|z - (-2 + i)| = 3$, so the centre is **(−2,** 1) and the radius is **3**. [3]
The sign trap is real: $|z + 2 - i|$ centres at $-2 + i$, not at $2 - i$.
The second says z is equidistant from 4 and from 2i, so the locus is the perpendicular bisector of the segment joining (4, 0) and (0, 2). That segment has gradient −1/2 and midpoint (2, 1), so the bisector has gradient 2 and equation **y** = 2x − 3. B1 B1 for the centre and radius, B1 for the equation of the bisector.

A2 It is a half-line starting at the point 2 on the real axis and running up and to the right at 45° to the positive real direction, that is the part of $y = x - 2$ with $x > 2$. The starting point itself is excluded, because the argument of zero is undefined. B1 for the half-line, B1 for the start at 2, B1 for the direction. An argument condition always gives a half line, never the whole line. [3]

SECTION B · Standard

B1 The argument of a quotient is the difference of the arguments, so the condition says the vector from 1 to z turns through a right angle from the vector from −1 to z. The angle subtended at z by the segment joining −1 and 1 is therefore 90°, and the angle in a semicircle puts z on the circle with that segment as diameter: **|z|** = 1. The sign of the argument selects the upper semicircle, and the two endpoints are excluded. M1 for the difference of arguments, A1 for the right angle at z, M1 for the angle in a semicircle, A1 for the upper semicircle with ends excluded. [4]

B2 The first condition is the disc of radius 4 centred at the origin, boundary included. The second says z is at least as far from 3 as from −3, which is the half-plane $x \le 0$. The region is the left half of the disc, a semicircular area of **8π**, about 25.13 square units. B1 for the disc, M1 A1 for the half-plane $x \le 0$, B1 for the area. [4]

B3 Write $z = x + iy$ and square both sides, which clears the roots in one step: $(x - 3)^2 + y^2 = 4[(x + 3)^2 + y^2]$. Squaring before expanding is the method mark; squaring term by term is the usual wrong turn. [5]
Expanding and collecting gives $3x^2 + 30x + 27 + 3y^2 = 0$, so $x^2 + 10x + 9 + y^2 = 0$.
Completing the square gives **(x** + 5)² + y² = 16, a **circle** of centre (−5, 0) and radius 4. M1 for squaring both sides, A1 for the expanded equation, M1 for completing the square, A1 for the circle equation, A1 for the centre and radius. A ratio of distances gives a circle whenever the ratio is not 1; only the ratio 1 produces a straight line.

SECTION C · Stretch

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C1 The circle has centre 4 and radius 2, and the origin lies outside it since $4 > 2$. [6]
Sketch it before anything else; the extremes are read off the diagram, not conjured from algebra.
For the modulus, the nearest and furthest points lie on the line through the origin and the centre, so the values are $4 - 2 =$ **2** and $4 + 2 =$ **6**.
For the argument, the extreme positions are where a line through the origin touches the circle. The radius to the point of contact is perpendicular to that tangent, giving a right-angled triangle with hypotenuse 4, the distance to the centre, and opposite side 2, the radius. So $\sin\theta = 2/4 = 1/2$ and $\theta = \pi/6$.
By symmetry in the real axis the greatest argument is **$\pi/6$** and the least is **$-\pi/6$**. The tangent length itself is $\sqrt{(4^2 - 2^2)} = 2\sqrt{3}$, which is worth quoting as evidence that the right-angled triangle was used.
Treating the argument like the modulus, and adding or subtracting the radius, is the mistake that this question is built to catch. B1 for the sketch, M1 A1 for the two moduli, M1 for the right-angled triangle, A1 for $\sin\theta = 1/2$, A1 for the two arguments.

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