InkMaths

QUESTION WORKBOOK

Further packing and the knapsack problem

Decision Mathematics 1 · Year FM

15 questions · 67 marks

NAVY SCREEN EDITION

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SECTION A · Warm-up

A1 FPK-A1

State what a knapsack problem asks for, and say how it differs from a bin packing problem. [2]

A2 FPK-A2

A van carries 12 kg. The crates are R at 4 kg for £36, S at 5 kg for £42, T at 7 kg for £56 and V at 8 kg for £60. Rank the crates by profit per kilogram and load the van by that ranking. [3]

A3 FPK-A3

A bag carries 9 kg. The items are F at 4 kg for £24, G at 6 kg for £30 and H at 5 kg for £20. Allowing items to be split, find the greatest profit the bag can carry, and state what that figure establishes about the best load of whole items. [3]

A4 FPK-A4

State the one version of the knapsack problem for which loading in order of profit per unit mass is certain to give the best answer, and say why the whole-item version escapes it. [2]

SECTION B · Standard

B1 FPK-B1 For the van in question A2, show that the greedy load is not the best, state the best load, and explain in one sentence why the ranking misled you. [4]

B2 FPK-B2 Formulate the van problem of question A2 as an integer programme. [3]

B3 FPK-B3 A bag carries 7 kg. The items are X at 2 kg for £14, Y at 3 kg for £19 and Z at 4 kg for £26. Build the dynamic programming table for limits 0 to 7 kg and use it to name the best load. [5]

SECTION C · Stretch

C1 FPK-C1

Six rectangles are to be cut from a roll 10 units wide, without being turned: A 4 by 6, B 3 by 6, C 3 by 4, D 5 by 4, E 5 by 4 and F 3 by 2, width given first. Pack them by shelves of decreasing height, find the area lower bound, and then find a packing that meets the bound. [8]

SECTION B · Standard

B4 FPK-B4

A van carries 12 kg. The crates are R at 4 kg for £36, S at 5 kg for £42, T at 7 kg for £56 and V at 8 kg for £60, and the best whole-crate load is known to be worth £98. Load the van by profit alone rather than by profit per kilogram. Then allow crates to be split and load in profit-per-kilogram order to obtain an upper bound, and explain what the bound does and does not tell you about the £98. [5]

B5 FPK-B5

A lorry rated at 1000 kg is to be loaded from 20 crates whose masses are whole numbers of kilograms. Compare the number of subsets a complete search would check with the number of cells in the dynamic programming table, and explain what happens to each count if the same lorry is rated at 1 000 000 g instead. [4]

B6 FPK-B6

A van carries 9 kg. The items are F at 3 kg for £21, G at 4 kg for £26 and H at 5 kg for £34. Build the dynamic programming table for limits 0 to 9 kg, use it to name the best load, and compare that load with the one profit per kilogram would have given. [6]

B7 FPK-B7

Five rectangles are to be cut from a roll 8 units wide, without being turned: J 5 by 4, K 3 by 4, L 4 by 3, N 4 by 3 and P 2 by 2, width given first. Pack them by shelves of decreasing height, state the length used, and find the area lower bound. [4]

SECTION C · Stretch

C2 FPK-C2

A container has a base 6 units by 4 and is open at the top. Six boxes are to be stacked in it without being turned: J with base 6 by 4 and height 2, K and L each with base 3 by 4 and height 3, M with base 6 by 2 and height 2, and N and P each with base 3 by 2 and height 2. Find a lower bound for the height of the stack, and find a stacking that meets it. [6]

C3 FPK-C3

A bag carries 10 kg. The items are R at 3 kg for £27, S at 4 kg for £34, T at 6 kg for £48 and V at 7 kg for £54. Load the bag by profit per kilogram, then by profit alone, then find the best load of whole items, and finally find the upper bound obtained by allowing items to be split. State what the four figures together establish. [7]

C4 FPK-C4

A knapsack carries 10 kg and the items are X (5 kg, £10), Y (5 kg, £10) and Z (10 kg, £21). Find the best load. Every mass and the limit are then doubled: explain why the best load does not change. Instead, £2 is added to the profit of every item, the masses being unchanged: find the new best load and explain why adding the same amount to every profit can change the answer. [5]

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