inkMaths

ANSWER BOOK

Game theory: play safe and stable solutions

Decision Mathematics 2 · Year FM

6 questions · 24 marks

NAVY EDITION

SECTION A · Warm-up

A1 The matrix is written from the **row** player's point of view, so if those choices are made the row player gains 5 and the column player loses 5. The game is zero-sum, so the column player is looking for **small** entries, not large ones. B1 for the row player's gain of 5, B1 for the column player's loss. [2]

A2 Maximin = max(4, 2) = 4 and minimax = min(7, 4, 9) = 4. They agree, so the game **is** stable with value **4**, occurring where the first row meets the second column. M1 for comparing maximin with minimax, A1 for the value. [2]

SECTION B · Standard

B1 Row minima are 2, 1 and 2, so the row player's maximin is **2**, playing **R1** or R3: both guarantee the same worst case. [6]
Column maxima are 4, 5 and 6, so the column player's minimax is **4**, playing **C1**.
Write the row minima beside the matrix and the column maxima beneath it. Those two lists are where the method marks are; a bare pair of strategies scores one mark at most.
Maximin 2 does not equal minimax 4, so the game is **not** stable and a mixed strategy is needed. All that has been established is that the value lies between 2 and 4. M1 A1 for the row minima and maximin, M1 A1 for the column maxima and minimax, A1 for the play-safe strategies, A1 for the game not being stable.

B2 Row minima are 3, 4 and 1, so the maximin is **4**, playing R2. [4]
Column maxima are 6, 4 and 8, so the minimax is **4**, playing C2.
They agree, so the game is **stable** with value **4**, at **R2** with C2. Neither player can improve by moving alone. Switching row loses the row player 1 or 3, and switching column costs the column player 1 or 4. M1 A1 for the maximin, M1 A1 for the minimax and the stable value.

B3 R2 beats R1 in every column, since 6 > 5, 4 > 3 and 8 > 7, so **R2** dominates R1. R2 also beats R3 everywhere, so **R2** dominates R3: delete both rows. [4]
In the single remaining row, the column player wants the smallest of 6, 4 and 8, so **C2** dominates C1 and C3 and both go.
The matrix reduces to the single entry **4**, so the game is stable with value 4 at R2 with C2. M1 A1 for the row dominance, M1 A1 for the column dominance and the value.
Dominance reaches the answer without any comparison of maximin with minimax.
Quote the inequalities that justify each deletion; 'looks worse' is not a justification.

SECTION C · Stretch

C1 Work in terms of k. The row minima are 0, min(4, k) and 1, so the maximin is **k** when $k \le 4$ and **4** when $k \ge 4$. [6]
The column maxima are 6, max(1, k) and 8, so the minimax is min(6, k) for $k \ge 1$, which is **k** when $k \le 6$ and **6** when $k \ge 6$.
Setting the two equal, they agree only while both equal k, that is for **k** = 1, 2, 3, 4, and then the value of the game is **k**, at R2 with C2. M1 A1 for the maximin in terms of k, M1 A1 for the minimax in terms of k, A1 for the four values of k, A1 for the value of the game. For k = 5 the maximin is 4 against a minimax of 5, and for $k \ge 6$ it is 4 against 6, so those games are not stable.
Note that maximin never exceeds minimax. The entry where the play-safe row meets the play-safe column is at least that row's minimum and at most that column's maximum, so the two agree only when that entry is smallest in its row and largest in its column, which is exactly a saddle point. Answers that produce a maximin above the minimax have an arithmetic error somewhere.