inkMaths

ANSWER BOOK

Geometric and negative binomial distributions

Further Statistics 1 · Year FM

7 questions · 24 marks

NAVY EDITION

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SECTION A · Warm-up

A1 The binomial fixes the number of trials and counts successes; the geometric fixes the number of successes at one and counts the trials needed to get it. B1 B1 for the two descriptions. The roles of the two quantities swap over. [2]

A2 $P(X = 4) = 0.3 \times 0.7^3 =$ **0.1029**: three failures then a success. The mean is $1/p =$ **3.33** trials. M1 for the product, A1 for 0.1029, B1 for the mean. The mode is 1 even though the average wait is longer than three, because the probabilities only ever decrease. [3]

SECTION B · Standard

B1 Needing at least five trials means the first four all failed, so $P(X \geq 5) = 0.7^4 =$ **0.2401**. The whole upper tail collapses to a single power, because 'at least x trials' and 'the first x - 1 all failed' describe the same event. M1 for the first four failing, A1 for 0.2401, B1 for the reason no summation is needed. Summing $0.3(0.7)^{x-1}$ from $x = 5$ upwards gives the same number and takes far longer. [3]

B2 This is negative binomial with $r = 2$ and $p = 0.25$, so $P(X = 6) = {}^5C_1 \times 0.25^2 \times 0.75^4 = 5 \times 0.0625 \times 0.3164 =$ **0.0989**. The mean is $r/p =$ **8** inspections, with variance $r(1 - p)/p^2 = 2(0.75)/0.0625 =$ **24**. M1 for the negative binomial form, A1 for 0.0989, B1 for the mean, B1 for the variance. [4]

B3 The final trial is pinned as a success by definition, so it is not free to move. Only the earlier $r - 1$ successes can be arranged, and only among the first $x - 1$ trials. Using ${}^xC_r$ would count arrangements in which the rth success arrives early, which describes a different event. B1 for the last trial being fixed, B1 for arranging the earlier successes only, B1 for what the wrong combination would count. [3]

B4 With $r = 1$ the combination is ${}^{x-1}C_0 = 1$, so $P(X = x) = p(1 - p)^{x-1}$, the geometric formula. The mean r/p becomes **$1/p$** and the variance $r(1 - p)/p^2$ becomes **$(1 - p)/p^2$**. Waiting for one success is the special case of waiting for r of them. B1 for the probability, B1 for the mean, B1 for the variance. [3]

SECTION C · Stretch

C1 Mean $r/p = 12$ and variance $r(1 - p)/p^2 = 24$. Dividing the second by the first removes r altogether: $(1 - p)/p = 24/12 = 2$. [6]
So $1 - p = 2p$ and $p =$ **1/3**. Then $r = 12p =$ **4**. Dividing the two moments is the move worth finding; solving the pair by substitution works but takes twice the algebra.
$P(X = 5) = {}^4C_3\, (1/3)^4\, (2/3) = 4 \times (1/81) \times (2/3) =$ **8/243**, about 0.0329.
For the tail, note that X cannot be less than $r = 4$, so only two values sit below 6. $P(X = 4) = {}^3C_3\, (1/3)^4 = 1/81 = 3/243$, with no arrangement to make, since all four trials are successes.
$P(X \geq 6) = 1 - 3/243 - 8/243 =$ **232/243**, about 0.955. Starting the tail at $X = 1$ rather than $X = r$ is the slip here, and it produces probabilities that do not sum to one.
M1 for dividing the variance by the mean, A1 for $p = 1/3$, A1 for $r = 4$, M1 A1 for 8/243, A1 for 232/243.