inkMaths

ANSWER BOOK

Geometric series

Sequences and series · Year 12–13

7 questions · 21 marks

NAVY EDITION

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SECTION A · Warm-up

A1 Each term is three times the one before, so $r = 3$ and $u_8 = 2 \times 3^7 = 4374$. B1 for the ratio, B1 for the eighth term. Seven multiplications reach the eighth term. The off-by-one is the same one arithmetic sequences carry, and $2 \times 3^8 = 13\,122$ is what it costs. [2]

A2 The sum to infinity exists when $|r| < 1$, so that the powers of r die away. Ratios 0.8 and −0.5 qualify; 1.2 does not, because its powers grow and the partial sums run off without settling. B1 for the condition, B1 for the three ratios. [2]

SECTION B · Standard

B1 $a = 5$ and $r = 2$, so $S_{12} = 5(2^{12} - 1)/(2 - 1) = 5 \times 4095 = 20\,475$. M1 for the sum formula, A1 for 5×4095, A1 for 20 475. With $r > 1$ the flipped dressing $a(r^n - 1)/(r - 1)$ keeps every bracket positive, which is worth the flip. [3]

B2 The ratio is $18/24 = 0.75$, comfortably inside the window, so the infinite sum exists and $S_\infty = 24/(1 - 0.75) = 96$. B1 for the ratio with the condition stated, M1 for the formula, A1 for 96. Endless additions still reach a finite total, because each one is a fixed fraction of the last and the leftovers shrink geometrically. State $|r| < 1$ before using the formula; that check is a mark in its own right. [3]

B3 Write $S_n = a + ar + \dots + ar^{n-1}$ and multiply by r: $rS_n = ar + ar^2 + \dots + ar^n$. Subtracting, every term cancels except the first of the first line and the last of the second: $S_n(1 - r) = a - ar^n$, and dividing by $1 - r$ finishes. The cancellation is the proof; showing it explicitly is what the marks are for. M1 for multiplying by r, A1 for the second line, M1 for subtracting, A1 for rearranging to the result. [4]

B4 After n years the balance is 2000×1.06^n, so exceeding 3000 needs $1.06^n > 1.5$. Taking logs of both sides gives $n > \log 1.5/\log 1.06 = 6.96$, and the first whole year past that mark is $n = 7$. Both sides confirm it. At $n = 6$ the multiplier is 1.419, still short of 1.5, and at $n = 7$ it is 1.504, past it. M1 for the inequality, A1 for $1.06^n > 1.5$, M1 for taking logs, A1 for seven years. A show-that question wants the inequality set up and the log step written out; a table of balances alone is worth about half the marks, and rounding 6.96 down to 6 throws away the rest. [4]

SECTION C · Stretch

C1 0.727272... = 0.72 + 0.0072 + 0.000072 + ..., a geometric series with a = 0.72 and r = 0.01. So the total is 0.72/(1 - 0.01) = 72/99 = 8/11. M1 for a = 0.72 and r = 0.01, A1 for the sum to infinity, A1 for 8/11. Every recurring decimal is an infinite geometric series in disguise, and that is the reason every one of them is a fraction. Writing 0.72 and 0.01 rather than 72 and 100 keeps the series honest. [3]