inkMaths

ANSWER BOOK

Goodness-of-fit tests

Further Statistics 1 · Year FM

7 questions · 26 marks

NAVY EDITION

SECTION A · Warm-up

A1 $\chi^2 = \Sigma(O - E)^2/E$. Dividing by E scales each gap against the size of the class it sits in, so a discrepancy of 3 where 5 were expected counts far more heavily than the same gap where 500 were expected. B1 for the statistic, B1 for the reason for the divisor. [2]

A2 H_0: the spinner is fair, so all five sectors are equally likely; H_1: it is not. [4]
Each expected frequency is 100/5 = **20**. The squared differences are 25, 4, 4, 25 and 0, totalling 58, and dividing by 20 gives χ^2 = **2.9**. B1 B1 for the two hypotheses, M1 for the contributions, A1 for the statistic. Set the observed and expected rows out in a table with a row of contributions underneath, since that is where the method marks are.

SECTION B · Standard

B1 Five classes with no parameter estimated: 5 - 1 = 4 degrees of freedom. Since 2.9 < 9.488, do not reject H_0: there is insufficient evidence at the 5% level that the spinner is biased. That is not proof of fairness, only an absence of evidence against it. B1 for the degrees of freedom, M1 for the comparison, A1 for the conclusion in context. [3]

B2 The estimate is the sample mean. The 60 components carry 0 × 12 + 1 × 18 + 2 × 15 + 3 × 9 + 4 × 6 = 99 flaws between them, so the estimate is 99/60 = **1.65**. [4]
That count is available only because the table stops at four. Had the last class read '4 or more', 99 would be a floor rather than a total, 1.65 would be a lower bound rather than the mean, and λ would have to be fitted by likelihood instead. Read the final class before you total anything.
For the test the classes are 0, 1, 2, 3 and '4 or more', so there are five of them. Subtract one because the expected frequencies are forced to total 60, and one more for the estimated λ, leaving **3** degrees of freedom. Forgetting the second subtraction is the standard error, and it makes the test too generous to the model. M1 for the total number of flaws, A1 for 1.65, M1 for counting the classes, A1 for three degrees of freedom.

B3 Any class with an expected frequency below 5 is combined with a neighbour, repeatedly if necessary, until every expected frequency reaches 5. Small expected values inflate the $(O - E)^2/E$ terms and distort the statistic. Pooling reduces the number of classes, and the degrees of freedom are counted from the pooled classes, not the original ones. B1 for the pooling rule, B1 for the reason, B1 for counting from the pooled classes. [3]

B4 Specifying the parameter in advance gives one more degree of freedom, since nothing has been estimated. Estimating it from the data lets the model bend towards the observations, so the expected frequencies sit closer to what was seen and the squared gaps shrink. Surrendering a degree of freedom is the price of that flexibility, and it keeps the test honest. B1 for which has more degrees of freedom, B1 for the model bending towards the data, B1 for the smaller statistic. [3]

SECTION C · Stretch

C1 H_0: a Poisson distribution is a suitable model for these data; H_1: it is not. [7]
With $\lambda = 1.65$, $P(X = 0) = 0.1920$, $P(X = 1) = 0.3169$, $P(X = 2) = 0.2614$ and $P(X = 3) = 0.1438$.
The model puts probability on 5, 6 and beyond as well, so its final class is '4 or more' and it takes whatever is left. The observed 6 components with four flaws sit against it, since none had more. Multiplying by 60 gives expected frequencies **11.52,** 19.01, 15.69, 8.63 and 5.15, which total 60 as they must.
Every expected frequency is above 5, so **no** pooling is needed.
Contributions $(O - E)^2/E$: 0.0197, 0.0540, 0.0300, 0.0161 and 0.1398, giving $\chi^2 =$ **0.260**.
Degrees of freedom = 5 - 1 - 1 = **3**, so the critical value is 7.815. Since $0.260 < 7.815$, **do** not reject H_0. The Poisson model fits these data well at the 5% level.
Two traps sit in this question. The model's last class must be '4 or more' so that the expected frequencies total 60, and the critical value must be read at 3 degrees of freedom rather than 4.
B1 for the hypotheses, M1 A1 for the expected frequencies, M1 A1 for the contributions and the statistic, B1 for the degrees of freedom, A1 for the conclusion.