

inkMaths

ANSWER BOOK

Graphs, proportion and transformations

Algebra and functions · Year 12–13

7 questions · 28 marks

NAVY EDITION

SECTION A · Warm-up

- A1** The simple factor gives a crossing at $(1, 0)$ and the squared factor gives a touch at $(-2, 0)$. Putting $x = 0$ gives $y = (0 - 1)(0 + 2)^2 = -4$, so the y -intercept is $(0, -4)$. B1 B1 for the three points. Squared factor means touch, simple factor means crossing, and the factorised form hands over the whole sketch without any expanding. [3]
- A2** Write $y = kx^2$. Then $12 = 4k$ gives $k = 3$, so $y = 3x^2$ and at $x = 5$, $y = 75$. M1 for the statement with a constant in it, A1 for $k = 3$, A1 for 75. Writing $y = 12x^2$ straight from the given pair is the standard error, and it comes from skipping the step where k is pinned down. [3]

SECTION B · Standard

- B1** Write $y = k/x$, so $k = 8 \times 3 = 24$ and at $x = 6$, $y = 4$. Doubling x halves y , which is the check worth doing in your head. The graph is the reciprocal curve, with asymptotes $x = 0$ and $y = 0$, the two axes themselves. M1 A1 for the model and k , A1 for $y = 4$, B1 for both asymptotes. Both asymptotes belong in a complete answer. [4]
- B2** Outside the bracket, the output doubles, so $(4, 3)$ becomes $(4, 6)$. Inside the bracket, $2x$ must equal 4, so $x = 2$ and the point becomes $(2, 3)$. B1, then M1 A1. Outside stretches vertically by scale factor 2; inside squashes horizontally by scale factor $\frac{1}{2}$. The inside move always runs backwards, which is the whole difficulty of the topic. [3]
- B3** The $x + 3$ inside translates the graph 3 to the left, and the factor 2 outside stretches it vertically by scale factor 2. Every point (a, b) therefore moves to $(a - 3, 2b)$. The maximum $(2, 5)$ goes to $(-1, 10)$. The crossings have $b = 0$, and doubling 0 changes nothing, so $(-1, 0)$ goes to $(-4, 0)$ and $(4, 0)$ goes to $(1, 0)$. B1 for the translation, B1 for the stretch, A1 for the maximum, A1 for both crossings. Shifting right instead of left is the error the mark scheme is watching for here. [4]

SECTION C · Stretch

- C1** Replacing x by $x - 2$ shifts the picture 2 to the right, and adding 3 lifts it 3 up, so the transformation is a translation by the vector $(2, 3)$. The asymptotes travel with it, moving from $x = 0$ and $y = 0$ to $x = 2$ and $y = 3$. For the y -axis crossing put $x = 0$: $y = 4/(-2) + 3 = 1$, giving $(0, 1)$. For the x -axis crossing put $y = 0$: $4/(x - 2) = -3$, so $x - 2 = -4/3$ and $x = 2/3$, giving $(2/3, 0)$. B1 for the translation, B1 B1 for the asymptotes, B1 for $(0, 1)$, M1 A1 for $(2/3, 0)$. The original curve never meets either axis at all; both crossings are created by the shift, so a candidate who answers 'none' has stopped thinking one step too early. [6]

- C2** Write $T = k\sqrt{L}$. Then $1 = k\sqrt{25} = 5k$, so $k = 1/5$ and $T = \sqrt{L}/5$. At $L = 81$, $T = 9/5 = 1.8$ [5]
seconds. To double T you must double $\sqrt{L}$, which means multiplying L by 4, an increase of 300%. M1 A1 for the model and k , A1 for 1.8, M1 for doubling the square root, A1 for 300%. Two traps sit in the last part. Doubling L only multiplies T by $\sqrt{2}$, and the answer wanted is the increase, 300%, not the final multiplier of 400%.
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