inkMaths

QUESTION WORKBOOK

Groups and their axioms

Further Pure 2 · Year FM

7 questions · 28 marks

PRINT EDITION

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SECTION A · Warm-up

A1 State the four axioms a set and a binary operation must satisfy to form a group. [3]

A2 Show that $\{1, 3, 5, 7\}$ under multiplication modulo 8 is a group, and state the order of each element. [4]

SECTION B · Standard

B1 Build the Cayley table for $\{1, 2, 3, 4\}$ under multiplication modulo 5, show the set is a group, and find a generator. [5]

B2 Explain the difference between the order of a group and the order of an element, using the symmetries of an equilateral triangle. [3]

B3 Show that the set of matrices with rows (1, n) and (0, 1), for integer n, forms a group under matrix multiplication, and say which familiar group it matches. [4]

B4 Explain why {0, 1, 2, 3} under multiplication modulo 4 is not a group, and why removing 0 does not repair it. [3]

SECTION C · Stretch

C1 G is a group in which every element x satisfies $x^2 = e$. Prove that G is abelian, and state a group of order 4 with this property. [6]