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ANSWER BOOK

Hooke's law and elastic strings

Further Mechanics 1 · Year FM

7 questions · 26 marks

NAVY EDITION

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SECTION A · Warm-up

A1 $T = \lambda x/l$, where T is the tension, x the extension beyond the natural length, l the natural length and λ the modulus of elasticity. The modulus is measured in newtons, since x/l is a ratio. B1 for the formula, B1 for the symbols, B1 for the units. [3]

A2 Extension = 2.5 - 2 = **0.5** m. T = 80 × 0.5/2 = **20** N. Using the length 2.5 in place of the extension gives 100 N, and that is the standard error in this topic. M1 for using the extension, A1 for 20 N. [2]

SECTION B · Standard

B1 Resolving vertically for the particle, the tension equals the weight, so T = 3(9.8) = **29.4** N. Hooke's law then gives $147x/1.2 = 29.4$, so $122.5x = 29.4$ and x = **0.24** m. The string is **1.44** m long. State T = W before using Hooke's law, since that equation carries a mark of its own. M1 for T = W, A1 for 29.4 N, M1 for Hooke's law, A1 for the extension and the total length. [4]

B2 The two natural lengths total 2.4 m, so the extensions add to 3 - 2.4 = 0.6 m. The particle is in equilibrium between two strings that both pull, so the tensions are equal. Then $24x_1/1.2 = 36x_2/1.2$, so $24x_1 = 36x_2$ and $x_1 = 1.5x_2$. Hence $2.5x_2 = 0.6$, giving x_2 = **0.24** m and x_1 = **0.36** m. The tension is 24(0.36)/1.2 = **7.2** N, and the other string gives the same. B1 for the extensions totalling 0.6 m, M1 for equating the tensions, A1 for $x_1 = 1.5x_2$, A1 for the two extensions, A1 for the tension. [5]

B3 A spring can be compressed as well as stretched, giving a thrust with the same formula and the compression in place of the extension, whereas a string goes slack and exerts nothing when its ends come closer than the natural length. A spring also stays straight in compression, so it can hold a particle away from a support, which a string cannot. B1 B1 for the two differences. [2]

B4 Resolving along the slope, the tension balances the component of weight down it: T = 2(9.8)sin30° = **9.8** N. The normal reaction is perpendicular to the slope and does not enter this equation. Then $49x/1 = 9.8$, so x = **0.2** m. M1 for resolving along the slope, A1 for the tension, M1 for Hooke's law, A1 for the extension. On a smooth plane no friction term appears. On a rough one the extension would depend on which way the particle was about to slip. [4]

SECTION C · Stretch

C1 The normal reaction is $R = 2(9.8) = 19.6$ N, so limiting friction is $\mu R = 0.2(19.6) =$ **3.92** N. [6]

The string pulls the particle towards O and friction is all that opposes it, so equilibrium requires $T \le 3.92$. Hooke's law gives $49x/1.5 \le 3.92$, that is $x \le$ **0.12** m.

A string exerts no force at all until it is stretched, so any distance up to the natural length also gives equilibrium. The particle can rest anywhere with **OP** ≤ 1.62 m, and the string is taut only for $1.5 < OP \le 1.62$.

Two points earn the marks here. Friction takes its limiting value only at the extreme position, so the condition is an inequality, and the slack case must be mentioned because a string cannot push the particle outwards.

B1 for $R = 19.6$ N, M1 A1 for the limiting friction, M1 for $T \le 3.92$, A1 for $x \le 0.12$ m, B1 for the slack case.