



QUESTION WORKBOOK

Induction: divisibility and matrices

Further proof · Year FM

7 questions · 30 marks

NAVY SCREEN EDITION

SECTION A · Warm-up

- A1** Let $f(n) = 8^n - 1$. Show that $f(1)$ is divisible by 7, and write down the inductive hypothesis for the claim that 7 divides $f(n)$ for all positive integers n . [2]

- A2** Using $f(n) = 8^n - 1$, simplify $f(k + 1) - 8f(k)$ and hence complete the divisibility proof. [3]

SECTION B · Standard

- B1** Prove by induction that $3^{2n} + 7$ is divisible by 8 for all positive integers n . [5]

- B2** Let M be the matrix with rows $(1, 1)$ and $(0, 1)$. Prove by induction that M^n has rows $(1, n)$ and $(0, 1)$. [5]

- B3** Let D be the matrix with rows $(2, 0)$ and $(0, 3)$. Prove by induction that D^n has rows $(2^n, 0)$ and $(0, 3^n)$. [5]

- B4** In divisibility proofs the working often forms $f(k + 1) - m f(k)$ for a chosen multiplier m . Explain how m is chosen, and why the manoeuvre works. [3]

SECTION C · Stretch

- C1** M is the matrix with rows $(3, -4)$ and $(1, -1)$. Prove by induction that M^n has rows $(1 + 2n, -4n)$ and $(n, 1 - 2n)$ for all positive integers n . [7]