inkMaths

ANSWER BOOK

Inequalities and inequations

Further Pure 1 · Year FM

6 questions · 25 marks

NAVY EDITION

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SECTION A · Warm-up

A1 $x - 4$ is negative when $x < 4$, and multiplying an inequality by a negative quantity reverses it, so one multiplication silently assumes a sign. The safe move is to subtract everything to one side and combine over a common denominator before factorising. B1 for $x - 4$ being negative when $x < 4$, B1 for the safe first step. [2]

A2 Subtract: $(x - 1 - 2(x + 2))/(x + 2) > 0$, that is $(-x - 5)/(x + 2) > 0$. Critical values -5 and -2. Testing the three intervals gives negative below -5, positive between, negative above -2. So $-5 < x < -2$, with both endpoints excluded. M1 for collecting on one side over a common denominator, A1 for the critical values, A1 for the solution set. [3]

SECTION B · Standard

B1 The right side must be positive, so $x > 0$ throughout. For $x \geq 3$ the modulus opens directly and $x - 3 < 2x$ gives $x > -3$, so all of $x \geq 3$ works. For $x < 3$ it opens with a sign change: $3 - x < 2x$ gives $x > 1$. Combining, the solution is $x > 1$. M1 for opening the modulus on $x \geq 3$, A1 for that branch, M1 for opening it with a sign change on $x < 3$, A1 for the combined solution. Squaring both sides would work here too, since both sides are positive on the region of interest. [4]

B2 Split at $x = \pm 2$, where the modulus changes. Where $|x| \geq 2$ the modulus opens directly and $x^2 - 3x - 4 > 0$, that is $(x - 4)(x + 1) > 0$, so $x < -1$ or $x > 4$; intersecting with $|x| \geq 2$ leaves $x \leq -2$ or $x > 4$. Where $|x| < 2$ the modulus opens with a sign change and $4 - x^2 > 3x$, that is $(x + 4)(x - 1) < 0$, so $-4 < x < 1$; intersecting with $-2 < x < 2$ leaves $-2 < x < 1$. Taking the union, **x** < 1 or $x > 4$. M1 for opening the modulus directly, A1 for $(x - 4)(x + 1) > 0$, A1 for that branch, M1 for opening it with a sign change, A1 for the second branch, A1 for the union. The two branches are combined with 'or', since a value only has to satisfy the inequality on the region it actually lives in; intersecting the branches instead loses everything below -1. [6]

B3 A zero of the numerator makes the expression exactly 0, which satisfies a weak inequality, so $x = -3$ is included. A zero of the denominator leaves the expression undefined, so $x = 2$ is excluded whatever the inequality sign. The solution is $x \leq -3$ or $x > 2$. B1 for including the numerator zero, B1 for excluding the denominator zero, B1 for the solution set. [3]

SECTION C · Stretch

C1 Subtract and combine: $(x(x - 1) - 3(x + 1))/((x + 1)(x - 1)) \le 0$, that is $(x^2 - 4x - 3)/((x + 1)(x - 1)) \le 0$. The numerator vanishes at $x = 2 \pm \sqrt{7}$ and the denominator at $x = \pm 1$, so there are four critical values. Testing the five intervals gives $-1 < x \le 2 - \sqrt{7}$ or $1 < x \le 2 + \sqrt{7}$. M1 for collecting over a common denominator, A1 for the single fraction, M1 for solving the numerator quadratic, A1 for $2 \pm \sqrt{7}$, B1 for the denominator critical values, M1 for testing the intervals, A1 for the solution set. The numerator zeros are included by the weak inequality; both denominator zeros stay out. [7]