INKMATHS FLASHCARDS

Decision Mathematics 2 allocation, flow and games

19 cards · fronts and backs

Print double-sided, flipping on the long edge, and each answer lands behind its question. Cut along the card borders.

Definitions in precise wording, the standard results the formulae booklet does not print, a check-yourself question per topic, and the slips that lose marks. The same cards run on the website with spaced-repetition scheduling, at inkphysics.co.uk/maths/flashcards.

DEFINITION

Define: North-west corner method.

RECALL

State the m + n - 1 rule for a transportation problem.

STANDARD RESULT

Improvement index:

index of an unused cell = ?

RECALL

Explain the stepping-stone loop.

DEFINITION

Define: Hungarian reduction.

RECALL

State the minimum lines test and what to do when it fails.

RECALL

Explain how to maximise an allocation.

DEFINITION

Define: Capacity of a cut.

RECALL

The number of cells a non-degenerate solution to an m by n transportation problem uses, which is exactly enough to fix the shadow costs. Fewer means degeneracy, fixed by recording a zero allocation.

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DEFINITION

Fill the top-left cell as far as supply and demand allow, cross off whichever runs out, repeat. It never looks at a cost, so the plan it produces is usually poor. Its job is to be feasible, not to be good.

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RECALL

From the entering cell, trace a closed route through cells in use, alternating plus and minus. The value moved is the smallest minus-cell allocation, that cell exits, and the cost falls by that value times the index.

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STANDARD RESULT

cost − R − K, where R + K = cost at every cell in use

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RECALL

If the fewest horizontal and vertical lines that cover every zero equals the size of the matrix, a complete allocation of zeros exists. If it is fewer, subtract the smallest uncovered entry from every uncovered entry and add it to every doubly covered one, then test again.

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DEFINITION

Subtract each row's minimum from that row, then each column's from that column. Every complete allocation loses the same total, so the cheapest one is unchanged and the zeros mark the bargains.

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DEFINITION

The total capacity of the arcs directed from the source side to the sink side only. Arcs crossing back towards the source contribute nothing at all.

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RECALL

Subtract every entry from the largest entry in the matrix, then minimise as usual. The biggest profit becomes the smallest cost, so the allocation found is the one of greatest total.

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STANDARD RESULT

Max-flow min-cut theorem:

maximum flow = ?

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DEFINITION

Define: Flow-augmenting path.

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DEFINITION

Define: Bellman's principle of optimality.

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RECALL

Explain play safe and stable solutions.

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DEFINITION

Define: Dominance.

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RECALL

Explain how to solve a mixed game.

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DEFINITION

Define: Expected monetary value.

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RECALL

State how to work back through a decision tree.

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DEFINITION

A route from source to sink using spare capacity forwards or existing flow backwards. It carries the smallest of those numbers along it, and at least one arc becomes saturated.

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STANDARD RESULT

capacity of the minimum cut

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RECALL

The row player takes the row whose worst entry is best, the maximin; the column player takes the column whose largest is smallest, the minimax. The game is stable exactly when the two agree, and that number is its value.

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DEFINITION

Any part of an optimal path is itself optimal between its own two ends. That is what licenses working backwards from the final stage, quoting values already found.

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RECALL

Play the first row with probability p, write the expected pay-off against each column as a line in p, and take the highest point of the lower boundary. That point is a crossing or an end, so equate the pair meeting there and test $p = 0$ and $p = 1$ as well. A peak at an end is a pure strategy, and a dominated row you should have deleted first.

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DEFINITION

A row is dominated when another is at least as good in every column; a column is dominated when another is at least as small in every row. Dominated options are deleted outright, often reducing a game to a two by two.

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RECALL

Start at the far right. A chance node takes the expected monetary value of its branches, a decision node takes the best figure available and you cross out the branches you rejected. Costs come off along the branch where they are actually met, not at the end.

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DEFINITION

The probability-weighted average of the outcomes reachable from a chance node, so the long-run average pay-off of taking that branch.

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SPOT THE ERROR

Say what is wrong with this claim:

The entering cell is the unused cell with the lowest cost in the table.

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SPOT THE ERROR

Say what is wrong with this claim:

Once every route from source to sink contains a full arc, the flow is maximal.

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CHECK YOURSELF

Why might a firm reject the option with the highest expected monetary value?

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SPOT THE ERROR

An augmenting path may run backwards along an arc that already carries flow, freeing capacity elsewhere. Only the absence of any augmenting path settles it, proved by a cut of the same capacity as the flow.

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SPOT THE ERROR

It is the cell with the most negative improvement index. The index accounts for what the rest of the plan must change, which the raw cost ignores, so a cell can be cheapest in the table and still have a positive index.

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CHECK YOURSELF

Expected monetary value is an average over many repetitions, and a one-off decision is not many repetitions. If the bad outcome would ruin the firm, a smaller certain gain is the better choice, which is what utility measures.

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