InkMaths

ANSWER BOOK

Integrating rational functions

Integration · Year 12–13

6 questions · 23 marks

NAVY EDITION

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SECTION A · Warm-up

A1 $(2/3)\ln|3x + 5| + c$. M1 for a logarithm of the bracket, A1 for the full answer. Differentiate it back and the chain rule's 3 cancels the 1/3, which is exactly what the divide-by-the-inner-coefficient step is for. Answers of $2\ln|3x + 5| + c$ ignore that step and score one mark at most. [2]

A2 $(5/2)\ln|2x - 1| + c$ and $3\ln|x + 2| + c$. M1 for the logarithmic form, A1 for the first integral, B1 for the second. A constant over a linear bracket always lands on a logarithm. The only arithmetic is the inner coefficient coming down as a divisor, and in the second integral that coefficient is 1, so nothing divides. [3]

SECTION B · Standard

B1 Split the fraction first. Writing $(3x + 5)/((x + 1)(x + 2)) = A/(x + 1) + B/(x + 2)$ and substituting the roots $x = -1$ and $x = -2$ gives $A = 2$ and $B = 1$. Each piece is a logarithm, so the integral is $2\ln|x + 1| + \ln|x + 2| + c$. M1 for setting up partial fractions, A1 for $A = 2$ and $B = 1$, M1 for integrating to logarithms, A1 for the answer. Nothing on the standard list covers the original fraction, and partial fractions exist for precisely this moment. [4]

B2 The numerator is half the derivative of the denominator, so the f'/f pattern applies and the integral is $\frac{1}{2}\ln(x^2 + 4) + c$. No modulus is needed, since $x^2 + 4$ is always positive. Partial fractions require the denominator to factorise into real linear brackets, and $x^2 + 4$ has discriminant -16, so it has no real roots. M1 for recognising the f'/f pattern, A1 for the integral, B1 for the reason partial fractions fail. Choosing the right tool is what this question tests. [3]

B3 Split first. $2/((x + 1)(x + 2)) = 2/(x + 1) - 2/(x + 2)$, so the integral is $[2\ln(x + 1) - 2\ln(x + 2)]$ from 0 to 1. That comes to $2(\ln 2 - \ln 3) - 2(\ln 1 - \ln 2) = 4\ln 2 - 2\ln 3 = 2\ln(4/3) = \ln(16/9)$. M1 for splitting into partial fractions, A1 for the two fractions, M1 for integrating to logarithms, A1 for substituting both limits, A1 for $\ln(16/9)$. The log laws do the tidying. A decimal answer of 0.575 throws away the exactness the question asked for, and forgetting the second bracket at the lower limit is the usual arithmetic slip. [5]

SECTION C · Stretch

C1 The fraction is improper, since numerator and denominator both have degree 2, so divide before splitting. The denominator expands to $x^2 + 3x + 2$, and the numerator is that plus 1, so the integrand is $1 + 1/((x + 1)(x + 2))$. Partial fractions on the remainder give $1/(x + 1) - 1/(x + 2)$. Integrating the three pieces gives $x + \ln|x + 1| - \ln|x + 2| + c$. M1 for dividing before splitting, A1 for the quotient plus remainder form, M1 for partial fractions on the remainder, A1 for the two fractions, M1 for integrating, A1 for the answer. Going straight to partial fractions without dividing produces an identity that cannot be satisfied, and the resulting answer loses the whole of the x term. Comparing degrees before anything else is the habit worth building. [6]