inkMaths

ANSWER BOOK

Intersections, angles and distances

Further vectors · Year FM

6 questions · 22 marks

NAVY EDITION

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SECTION A · Warm-up

A1 Parametrise: $x = 2 + \lambda$, $y = 2\lambda$, $z = 1 + 2\lambda$. Substituting: $(2 + \lambda) + 2\lambda + (1 + 2\lambda) = 13$, so $5\lambda + 3 = 13$ and $\lambda = 2$. The point is $(4, 4, 5)$, and $4 + 4 + 5 = 13$ confirms it. M1 for substituting the parametrisation into the plane, A1 for $\lambda = 2$, A1 for the point. [3]

A2 Line and plane: $\sin\theta = |b\cdot n|/(|b||n|)$ with the line's direction and the plane's normal. Two planes: $\cos\theta = |n_1\cdot n_2|/(|n_1||n_2|)$ with the two normals. B1 for the sine formula, B1 for the cosine formula. Sine for the first because the normal stands at right angles to the plane. [2]

SECTION B · Standard

B1 $b\cdot n = 1 + 2 + 2 = 5$, $|b| = 3$ and $|n| = \sqrt{3}$. So $\sin\theta = 5/(3\sqrt{3}) = 5\sqrt{3}/9$, giving $\theta = 74.2°$. M1 for $b\cdot n$ with both moduli, M1 for using sine, A1 for $5\sqrt{3}/9$, A1 for the angle. Using cosine by habit would give the complement 15.8°, the angle to the normal instead of to the plane. [4]

B2 Distance $= |2 - 2 + 6 - 3|/\sqrt{(4 + 1 + 4)} = 3/3 = 1$. M1 for substituting the point into the plane expression, M1 for dividing by the length of the normal, A1 for the distance. Substitute the point, subtract d, divide by the normal's length, and keep the modulus. [3]

B3 Normals $(1, 0, 1)$ and $(0, 1, 1)$ have dot product 1 and lengths $\sqrt{2}$ and $\sqrt{2}$. $\cos\theta = 1/2$, so $\theta = 60°$. M1 for both normals, M1 for the scalar product formula, A1 for $\cos\theta = 1/2$, A1 for the angle. The planes' angle is their normals' angle; no point of intersection is ever needed. [4]

SECTION C · Stretch

C1 The direction is $b = (1, -1, 0)$ and the normal is $n = (1, 1, 1)$, so $b\cdot n = 1 - 1 + 0 = 0$. The line runs at right angles to the normal, which puts it parallel to the plane, either inside it or clear of it. Testing the anchor point $(0, 0, 5)$ gives $0 + 0 + 5 = 5 \neq 1$, so the line misses the plane and one substituted point has settled which case holds. Because every point of the line is the same distance from the plane, that distance can be measured from the anchor: $|0 + 0 + 5 - 1|/\sqrt{(1 + 1 + 1)} = 4/\sqrt{3} =$ $\mathbf{4\sqrt{3}/3}$, about 2.31. M1 for $b\cdot n$, A1 for the zero showing the line is parallel, M1 for testing the anchor point, A1 for concluding it does not lie in the plane, M1 for the distance formula, A1 for the exact distance. Substituting the line into the plane gives $\lambda + (-\lambda) + 5 = 1$, that is $5 = 1$. The λ terms cancel and no solution survives, which is the algebraic signature of a line parallel to a plane it does not lie in. [6]