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ANSWER BOOK

Kinematics with constant acceleration

Mechanics · Year 12–13

8 questions · 36 marks

NAVY EDITION

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SECTION A · Warm-up

A1 Displacement s, initial velocity u, final velocity v, acceleration a and time t. The equation without s is $v = u + at$. B1 for the list, B1 for the equation. Each of the four standard equations omits exactly one quantity, so choosing by the letter that never appears in the question is the whole routine. [2]

A2 Taking $u = 0$, $v = u + at = 0 + 2 \times 6 = 12$ m s^{-1}, and $s = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2} \times 2 \times 36 = 36$ m. M1 A1 for the speed, M1 A1 for the distance. As a check, the average speed over the six seconds is $(0 + 12)/2 = 6$ m s^{-1} and $6 \times 6 = 36$ m, which agrees. 'From rest' is what supplies $u = 0$, and it is stated in words rather than in symbols on purpose. [4]

SECTION B · Standard

B1 Time is not mentioned in the first part, so use $v^2 = u^2 + 2as$: $0 = 900 + 300a$, giving $a = -3$ m s^{-2}, a deceleration of 3 m s^{-2}. Then $v = u + at$ gives $0 = 30 - 3t$, so $t = 10$ s. M1 A1 for the acceleration, M1 A1 for the time. The negative sign is doing real work here, since the acceleration opposes the motion, and an answer of $a = -3$ quoted as a deceleration of -3 m s^{-2} says the car is speeding up. [4]

B2 Taking downwards as positive, $u = 0$ and $a = 9.8$. Depth: $s = \frac{1}{2}gt^2 = \frac{1}{2} \times 9.8 \times 6.25 = 30.625$, so 30.6 m to 3 significant figures. Impact speed: $v = gt = 9.8 \times 2.5 = 24.5$ m s^{-1}. M1 A1 for the depth, M1 A1 for the speed. 'Dropped' supplies $u = 0$ without saying so, and reading the starting conditions out of the wording is half of every kinematics question. [4]

B3 Take up as positive, so $a = -9.8$ throughout. At the highest point $v = 0$, and $v^2 = u^2 + 2as$ gives $0 = 196 - 19.6s$, so $s = 10$ m. Time to the top is $14/9.8 = 1.43$ s, and by symmetry the whole flight takes twice that, 2.86 s. M1 A1 for the height, M1 A1 for the time. Fixing the positive direction once at the start and never revisiting it is what keeps the signs consistent, and mixing a positive g with an upward positive direction is the classic route to a negative height. [4]

B4 With up positive, $5 = 14t - 4.9t^2$, which rearranges to $4.9t^2 - 14t + 5 = 0$. The discriminant is $196 - 98 = 98$, so $t = (14 \pm \sqrt{98})/9.8$, giving $t = 0.418$ s and $t = 2.44$ s. M1 for forming the quadratic, M1 for solving it, A1 A1 for the two roots, B1 for the explanation. Two roots mean two visits, since the ball passes 5 m on the way up and again on the way down, and a candidate who discards the second root as 'extra' loses two marks for a physically real answer. [5]

SECTION C · Stretch

C1 Measure t from the first projection, with up positive. The first ball is at $h_1 = 14t - 4.9t^2$. The second has been in the air for $t - 1$ seconds, so $h_2 = 14(t - 1) - 4.9(t - 1)^2$. Setting them equal, the t^2 terms cancel and expanding leaves $0 = -14 + 9.8t - 4.9$, so $9.8t = 18.9$ and $t = 1.93$ s. Substituting back, $h = 14(1.9286) - 4.9(1.9286)^2 = 8.78$ m. M1 for the first height, M1 for using $t - 1$ in the second, M1 for equating, A1 for the linear equation, A1 for t, A1 for the height. The whole question turns on the shifted time. Writing 14t for the second ball as well makes the two expressions identical and the equation unsolvable. Both balls land at $t = 2.86$ s and $t = 3.86$ s respectively, so 1.93 s is genuinely during the flight of both. [6]

C2 Deal with the two phases separately. Phase one: $v = u + at$ gives $30 = 0 + 2.5t$, so the motorcycle reaches 30 m s^{-1} after 12 s, having covered $s = \frac{1}{2} \times 2.5 \times 12^2 = 180$ m. In the same 12 s the car has covered $20 \times 12 = 240$ m, so it is still 60 m ahead. Phase two: both now travel at constant speed, and the motorcycle closes at $30 - 20 = 10$ m s^{-1}, so it takes a further $60/10 = 6$ s. The motorcycle draws level 18 s after the car passed, having travelled $180 + 30 \times 6 = 360$ m, which matches the car's $20 \times 18 = 360$ m. M1 A1 for the 12 s, M1 A1 for the two distances, M1 for the closing speed, A1 for 18 s, A1 for 360 m. Suvat applies to each phase separately but never across the join, since the acceleration changes at $t = 12$. Treating the whole 18 s as one uniformly accelerated stretch is the mistake this question is built to expose. [7]