



QUESTION WORKBOOK

Leibnitz's theorem and the Weierstrass substitution

Further Pure 1 · Year FM

7 questions · 29 marks

PRINT EDITION

SECTION A · Warm-up

A1 State Leibnitz's theorem for the n th derivative of a product fg . [2]

A2 Under the substitution $t = \tan(x/2)$, write down $\sin x$, $\cos x$ and dx in terms of t . [3]

SECTION B · Standard

B1 Use Leibnitz's theorem to find the fourth derivative of $y = x^3e^x$. [4]

B2 Use $t = \tan(x/2)$ to evaluate $\int 1/(1 + \cos x) \, dx$ from 0 to $\pi/2$, exactly. [4]

- B3** Use $t = \tan(x/2)$ to show that $\int \sec x \, dx = \ln|(1+t)/(1-t)| + c$, and hence evaluate $\int \sec x \, dx$ from 0 to $\pi/3$, giving your answer in the form $\ln k$ with k exact. [5]

SECTION C · Stretch

- C1** Use Leibnitz's theorem to show that the n th derivative of $y = x^2 e^{2x}$ is $e^{2x}(2^n x^2 + n 2^n x + n(n-1)2^{n-2})$ for $n \geq 2$. [5]

- C2** Use the substitution $t = \tan(x/2)$ to evaluate $\int 1/(5 - 3\cos x) \, dx$ from 0 to $\pi/2$, giving your answer in exact form. [6]