InkMaths

QUESTION WORKBOOK

# Leibnitz's theorem and the Weierstrass substitution

Further Pure 1 · Year FM

7 questions · 29 marks

NAVY SCREEN EDITION


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## SECTION A · Warm-up

**A1** State Leibnitz's theorem for the nth derivative of a product fg. [2]

**A2** Under the substitution $t = \tan(x/2)$, write down $\sin x$, $\cos x$ and $dx$ in terms of $t$. [3]

## SECTION B · Standard

**B1** Use Leibnitz's theorem to find the fourth derivative of $y = x^3 e^x$. [4]

**B2** Use $t = \tan(x/2)$ to evaluate $\int 1/(1 + \cos x)\, dx$ from 0 to $\pi/2$, exactly. [4]

**B3** Use $t = \tan(x/2)$ to show that $\int \sec x \, dx = \ln|(1 + t)/(1 - t)| + c$, and hence evaluate $\int \sec x \, dx$ from 0 to $\pi/3$, giving your answer in the form $\ln k$ with $k$ exact. [5]

## SECTION C · Stretch

**C1** Use Leibnitz's theorem to show that the nth derivative of $y = x^2e^{2x}$ is $e^{2x}(2^n x^2 + n2^n x + n(n - 1)2^{n-2})$ for $n \geq 2$. [5]

**C2** Use the substitution $t = \tan(x/2)$ to evaluate $\int 1/(5 - 3\cos x)\,dx$ from 0 to $\pi/2$, giving your answer in exact form. [6]