inkMaths

ANSWER BOOK

Lines and planes in three dimensions

Further vectors · Year FM

7 questions · 26 marks

NAVY EDITION

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SECTION A · Warm-up

A1 Direction PQ = $(2, 1, 3)$, so $r = (2, 1, 0) + \lambda(2, 1, 3)$. B1 for the direction, B1 for a complete vector equation. Any point of the line and any non-zero multiple of the direction give an equally correct equation. [2]

A2 From x: $2 + 2\lambda = 8$, so $\lambda = 3$. Then $y = 1 + 3 = 4$ and $z = 0 + 9 = 9$, both matching. All three coordinates agree at the same λ, so the point is on the line. M1 for finding λ from one coordinate, M1 for testing the other two, A1 for the conclusion. [3]

SECTION B · Standard

B1 Cartesian: $2x - y + 2z = 9$. At the point: $6 - 1 + 4 = 9$, so yes, it lies in the plane. B1 for the cartesian equation, M1 for substituting the point, A1 for the conclusion. The scalar product form and the cartesian form are the same statement in different notation. [3]

B2 In-plane directions: AB = $(1, 2, 0)$ and AC = $(2, 0, 3)$. A normal (a, b, c) needs $a + 2b = 0$ and $2a + 3c = 0$; choosing $b = 1$ gives $a = -2$, $c = 4/3$, and scaling by 3 gives $n = (-6, 3, 4)$. Then $d = n \cdot A = 1$, so $-6x + 3y + 4z = 1$. Checking B and C: $-12 + 9 + 4 = 1$ and $-18 + 3 + 16 = 1$. M1 for two in-plane directions, M1 for the perpendicularity equations, A1 for the normal, M1 for finding the constant, A1 for the cartesian equation. [5]

B3 Solve each coordinate for λ: $(x - 2)/2 = (y - 1)/1 = z/3$. Each fraction equals λ, so setting them equal states that one value of the parameter produces all three coordinates at once, which is exactly membership of the line. M1 for solving each coordinate for λ, A1 for the cartesian form, B1 for the explanation. [3]

B4 The x-component of the direction is zero, so $(x - a)/0$ is meaningless. What it is trying to say is that x never changes. Write $x = a$ separately and set the remaining fractions equal, giving $x = a$, $(y - b)/2 = (z - c)/5$. B1 for the zero denominator being meaningless, B1 for x being constant, B1 for the correct form. A zero denominator signals a frozen coordinate rather than a formula to force. [3]

SECTION C · Stretch

C1 The directions are not multiples of each other, so the lines are not parallel. [7]
Equating components gives $1 + \lambda = 2$, $2 = \mu$ and $3 - \lambda = 1 + \mu$. The first two give $\lambda = 1$ and $\mu = 2$, but then the third reads $2 = 3$, so the lines never meet and are skew. A vector perpendicular to both is the cross product $(1, 0, -1) \times (0, 1, 1) = (1, -1, 1)$, of length $\sqrt{3}$. Take the vector between the two anchor points, $(2, 0, 1) - (1, 2, 3) = (1, -2, -2)$, and project it onto that common perpendicular. The shortest distance is $|(1, -2, -2)\cdot(1, -1, 1)|/\sqrt{3} = |1 + 2 - 2|/\sqrt{3} = 1/\sqrt{3} = \boldsymbol{\sqrt{3}/3}$, about 0.577. B1 for the directions not being parallel, M1 for equating components, A1 for the contradiction showing the lines are skew, M1 for the cross product, A1 for the common perpendicular, M1 for projecting the vector between the anchors, A1 for the shortest distance. Any pair of points, one on each line, gives the same projection, so the choice of anchors does not matter.