InkMaths

QUESTION WORKBOOK

Locating roots and iteration

Numerical methods · Year 13

8 questions · 22 marks

PRINT EDITION

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SECTION A · Warm-up

A1 $f(x) = x^3 + x - 5$. Show that $f(x) = 0$ has a root between $x = 1$ and $x = 2$. [2]

A2 Explain why a change of sign of f between a and b does not always mean a root of $f(x) = 0$ lies in (a, b), and give the condition that repairs the argument. [2]

A3 Show that the equation $x^3 + x - 5 = 0$ can be rearranged into the iterative form $x = (5 - x)^{1/3}$. [2]

SECTION B · Standard

B1 Using the iteration $x_{n+1} = (5 - x_n)^{1/3}$ with $x_0 = 1.5$, find x_1, x_2 and x_3, giving each to 5 decimal places. [3]

B2 Show that $\alpha = 1.516$ is a root of $x^3 + x - 5 = 0$ correct to 3 decimal places. [3]

B3 The cobweb diagram for the iteration $x_{n+1} = (5 - x_n)^{1/3}$ spirals inwards towards the root $\alpha \approx 1.516$. State the feature of $g(x) = (5 - x)^{1/3}$ near the root that guarantees convergence. [2]

SECTION C · Stretch

C1 The equation $x^3 + x - 5 = 0$ also rearranges to $x = 5 - x^3$. Starting from $x_0 = 1.5$, calculate x_1 and x_2, and explain what happens to this iteration. [4]

C2 An iteration $x_{n+1} = g(x_n)$ converges to a value α. Explain why α must satisfy the equation $x = g(x)$, and why a sequence that appears to settle on a calculator display is not by itself a proof that α is accurate to the digits shown. [4]