inkMaths

ANSWER BOOK

Logarithms and their laws

Exponentials and logarithms · Year 12

7 questions · 23 marks

NAVY EDITION

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SECTION A · Warm-up

A1 A logarithm is an exponent. Since $2^5 = 32$ the first is 5, since $5^3 = 125$ the second is 3, and since $10^{-2} = 0.01$ the third is -2. B1 B1 B1, one for each value. That last one trips people. Logs of numbers below 1 are negative, not impossible, and writing "undefined" loses the mark outright. [3]

A2 Combine left to right. log 8 + log 5 = log 40, then log 40 - log 10 = log (40/10) = log 4. M1 for one correct use of a log law, A1 for log 4. Addition multiplies and subtraction divides, which is the index laws wearing log clothing. Writing log 8 + log 5 as log 13 is the error the first law exists to kill. [2]

SECTION B · Standard

B1 The multiple moves inside first, so ½ log 49 = $\log 49^{1/2}$ = log 7. Then log 7 + log 2 = log 14. M1 for taking the multiple inside, A1 for log 7, A1 for log 14. The third law handles fractional multipliers exactly as it handles whole ones, and a half becomes a square root. Doing the addition before the halving gives ½ log 98, which is a different number. [3]

B2 Take ln of both sides to get 2x - 1 = ln 12, so x = (ln 12 + 1)/2 = 1.742 to 4 significant figures. M1 for taking logs, A1 for 2x - 1 = ln 12, A1 for 1.742. The log peels the exponential off, and everything after that is linear algebra. Keep ln 12 exact until the last line; rounding it to 2.48 first can shift the fourth figure. [3]

B3 Apply e to both sides, giving $2x + 5 = e^3$, so $x = (e^3 - 5)/2$ = 7.543 to 4 significant figures. M1 for applying e to both sides, A1 for $2x + 5 = e^3$, A1 for 7.543. Here the exponential has to be applied rather than peeled off, and deciding which way round is the real skill. Note that e^3 undoes the whole ln bracket at once; "$e^3 = 2x$, then + 5" is the misstep. [3]

B4 The bases refuse to match, so take logs of both sides and bring the power down: (2x + 1) log 5 = log 40. Then log 40/log 5 = 2.292, so 2x + 1 = 2.292 and x = 0.6460 to 4 significant figures. M1 for taking logs, A1 for (2x + 1) log 5 = log 40, M1 for rearranging, A1 for 0.6460. Any base of logarithm serves, because it cancels in the ratio. What does not work is log 40/log 5 = log 8; a quotient of logs is not the log of a quotient. [4]

SECTION C · Stretch

C1 Let $u = 2^x$; since $2^{2x} = u^2$, the equation is $u^2 - 5u + 4 = 0$, which factorises as $(u - 1)(u - 4) = 0$. So $2^x = 1$, giving $x = 0$, or $2^x = 4$, giving $x = 2$. M1 for the substitution, A1 for the quadratic in u, M1 for solving it, A1 A1 for the two values of x. A quadratic wearing an exponential disguise: the substitution unlocks it, and both roots here convert back cleanly. [5]