inkMaths

ANSWER BOOK

Matrix algebra and transformations

Matrices · Year FM

7 questions · 27 marks

NAVY EDITION

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SECTION A · Warm-up

A1 Rotation: rows (0, -1) and (1, 0). Reflection in $y = x$: rows (0, 1) and (1, 0). B1 B1 for the two matrices. Both can be read off from where (1, 0) and (0, 1) land, since the images are the columns. [2]

A2 AB has rows (4, -2), (6, 0); BA has rows (2, -2), (4, 2). They differ, because matrix multiplication is not commutative, so the order of a product must never be swapped mid-working. M1 for a correct multiplication method, A1 for both products, B1 for the comment on commutativity. [3]

SECTION B · Standard

B1 Multiply: rows (0, -1), (1, 0) applied to (3, 1) give (-1, 3). B1 for the rotation matrix, M1 for the multiplication, A1 for (-1, 3). A sketch confirms it. The point swings from the first quadrant into the second, a quarter turn against the clock. [3]

B2 The reflection acts first, so it sits on the right of the product. RM is rows (0, -1), (1, 0) times rows (1, 0), (0, -1), which gives rows (0, 1), (1, 0). That is the reflection in $y = x$. B1 for the reflection matrix, M1 for multiplying in the order RM, A1 for rows (0, 1), (1, 0), A1 for naming the reflection in $y = x$. Writing MR instead gives rows (0, -1), (-1, 0), the reflection in $y = -x$, so the order genuinely matters. [4]

B3 The images of the base vectors are the columns, giving rows (2, 1), (0, 3). Then (2, 5) maps to $(2 \times 2 + 1 \times 5, 0 + 3 \times 5) = (9, 15)$. B1 for the matrix, M1 for the multiplication, A1 for (9, 15). Columns-are-images turns matrix building into reading the question. [3]

B4 Squaring gives rows (-1, 0), (0, -1), which is $-I$, a half turn. Squaring again gives $(-I)^2 = I$, so $R^4 = I$. M1 for squaring, A1 for $-I$, M1 for squaring again, A1 for the fourth power being the identity. Four quarter turns make a full turn, and the matrix records that as its fourth power being the identity. [4]

SECTION C · Stretch

C1 Build each matrix from the images of the base vectors. R fixes i, sends j to k and k to -j, so R has rows (1, 0, 0), (0, 0, -1), (0, 1, 0). S fixes k, sends i to j and j to -i, so S has rows (0, -1, 0), (1, 0, 0), (0, 0, 1). S first means S on the right, so the combined matrix is RS, with rows **(0,** -1, 0), (0, 0, -1), (1, 0, 0). Applying it to (1, 2, 3) gives **(-2,** -3, 1). M1 for building a matrix from the images of the base vectors, A1 for R, A1 for S, M1 for the product in the order RS, A1 for RS, A1 for (-2, -3, 1), M1 for reversing the order, A1 for SR differing. Reversing the order gives SR, with rows (0, 0, 1), (1, 0, 0), (0, 1, 0), which sends (1, 2, 3) to (3, 1, 2). The two matrices differ, so the transformations differ. Rotations about different axes generally do not commute, and these two 90° rotations give different products, which is easy to confirm with a die or a book held at arm's length. [8]