



QUESTION WORKBOOK

Maximum flow and the labelling procedure

Decision Mathematics 2 · Year FM

7 questions · 26 marks

PRINT EDITION

SECTION A · Warm-up

- A1** Explain what the two numbers written on each arc during the labelling procedure represent. [2]

- A2** A flow-augmenting path has spare capacities 9, 4 and 6 along it. State the extra flow it carries and what happens to one of its arcs. [2]

SECTION B · Standard

- B1** A network has arcs SA 12, SB 9, AC 7, AD 6, BD 8, CT 10 and DT 11, with S the source and T the sink. Starting from zero flow, find the maximum flow, listing each flow-augmenting path with the flow it carries. [6]

- B2** In a network with source S and sink T, the arcs SA 12, SB 9, AC 7, AD 6, BD 8, CT 10 and DT 11 carry a flow of 18, made up of SA 12, SB 6, AC 7, AD 5, BD 6, CT 7 and DT 11. Prove that this flow is maximal, naming the theorem you use. [4]

- B3** A student argues that once every route from source to sink contains a saturated arc, the flow must be maximal. Explain the error. [2]

- B4** A network with source S and sink T has arcs SA 12, SB 9, AC 7, AD 6, BD 8, CT 10 and DT 11, and its maximum flow is 18. A new rule requires at least 4 units to travel along BD. State how the labelling procedure is started and whether 18 is still attainable. [3]

SECTION C · Stretch

- c1** A network has source S, sink T and arcs SA 6, SB 5, AB 3, AT 4 and BT 6. A student finds the flow SA 6, SB 3, AB 3, AT 3, BT 6, of value 9, and claims it is maximal because every route from S to T contains a saturated arc. Find a flow-augmenting path, state the flow it carries, give the maximum flow, and prove it is maximal.

[7]