InkMaths

QUESTION WORKBOOK

Mean values and improper integrals

Further calculus · Year FM

7 questions · 26 marks

PRINT EDITION

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SECTION A · Warm-up

A1 Define the mean value of a function f on the interval [a, b], and describe what it represents geometrically. [2]

A2 Find the mean value of $f(x) = \sin x$ on the interval $[0, \pi]$. [3]

SECTION B · Standard

B1 Find the exact mean value of $f(x) = e^{2x}$ on $[0, 1]$. [4]

B2 Evaluate $\int x^{-3/2}\,dx$ from 1 to ∞, or show that it diverges. [4]

B3 Evaluate $\int 1/\sqrt{x}\,dx$ from 0 to 4, treating the lower limit with care. [4]

B4 Both $1/x$ and $1/x^{3/2}$ tend to zero as $x \to \infty$, yet only one of $\int$ from 1 to ∞ converges. Identify which, and explain what distinguishes them. [3]

SECTION C · Stretch

C1 Show that $\int \ln x\,dx$ from 0 to 1 converges, and find its exact value. You may assume that $t \ln t \to 0$ as $t \to 0^+$. [6]