inkMaths

ANSWER BOOK

Modulus, argument and loci

Complex numbers · Year FM

7 questions · 26 marks

NAVY EDITION

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SECTION A · Warm-up

A1 $|z| = \sqrt{(1 + 3)} = 2$. The point $(-1, \sqrt{3})$ sits in the second quadrant, so $\arg z = \pi - \pi/3 = 2\pi/3$. B1 for the modulus, M1 for the quadrant, A1 for the argument. A sketch first; the calculator's arctan alone would report the wrong quadrant. [3]

A2 $x = 2 \cos(\pi/3) = 1$ and $y = 2 \sin(\pi/3) = \sqrt{3}$, so $z = 1 + \sqrt{3}\,i$. M1 for $x = r \cos \theta$ and $y = r \sin \theta$, A1 for $1 + \sqrt{3}\,i$. Modulus-argument to cartesian is two right-triangle reads. [2]

SECTION B · Standard

B1 Multiplying multiplies moduli and adds arguments: $|zw| = 6$, $\arg zw = \pi/6 + \pi/4 = 5\pi/12$. Dividing divides and subtracts: $|z/w| = 2/3$, $\arg(z/w) = \pi/6 - \pi/4 = -\pi/12$. B1 B1 for the modulus and argument of zw, B1 B1 for those of z/w. No cartesian expansion is needed anywhere. [4]

B2 A circle of radius 2 centred at $3 + 4i$, the point $(3, 4)$. The centre is distance 5 from the origin, so $|z|$ runs from $5 - 2 = 3$ to $5 + 2 = 7$, along the line joining the origin to the centre. B1 for the centre, B1 for the radius, M1 for the distance of the centre from the origin, A1 for the two values. Centre distance plus and minus radius, once the sketch is drawn. [4]

B3 Equal distance from 2 and from -2 gives the perpendicular bisector of the segment joining $(2, 0)$ and $(-2, 0)$, which is the imaginary axis. B1 for equal distances, B1 for the perpendicular bisector, B1 for naming the imaginary axis. Any equation $|z - a| = |z - b|$ is the perpendicular bisector of a and b. [3]

B4 $|z| \le 3$ is the disc of radius 3, and the argument condition keeps the first quadrant, including both bounding axes. The region is a quarter disc of area $(1/4)\pi(3^2) = 9\pi/4$, about 7.07. B1 for the disc, B1 for the argument region, M1 for a quarter of the disc area, A1 for $9\pi/4$. Sketch both loci and read the overlap; the algebra adds nothing here. [4]

SECTION C · Stretch

C1 Put $z = x + iy$ and square both sides, which is safe because both are non-negative. Then $(x - 2)^2 + y^2 = 4[(x + 1)^2 + y^2]$. Expanding gives $x^2 - 4x + 4 + y^2 = 4x^2 + 8x + 4 + 4y^2$, so $0 = 3x^2 + 3y^2 + 12x$. Dividing by 3 gives $x^2 + y^2 + 4x = 0$, and completing the square gives $(\mathbf{x} + 2)^2 + y^2 = 4$, a circle of centre $(\mathbf{-2}, 0)$ and radius **2**. M1 for putting $z = x + iy$ and squaring, A1 for the expanded equation, M1 for collecting terms, A1 for $x^2 + y^2 + 4x = 0$, M1 for completing the square, A1 for the centre and radius. An unequal ratio of distances always produces a circle, in contrast with $|z - a| = |z - b|$, where the squared terms cancel and a straight line is left. The centre is not the midpoint of 2 and -1, so it cannot be written down by inspection. [6]