inkMaths

ANSWER BOOK

Motion in a vertical circle

Further Mechanics 2 · Year FM

6 questions · 23 marks

NAVY EDITION

inkphysics.co.uk/maths/ · youtube.com/@InkPhysics

SECTION A · Warm-up

A1 The tension always acts along the string, which is perpendicular to the direction of motion at every instant, so it does no work. Only gravity does work, and it is conservative, so the total of kinetic and potential energy is constant. B1 for the tension doing no work, B1 for gravity being the only force that does work. [2]

A2 At the critical speed the string is just taut, so $T = 0$ and the only radial force is the weight: $mg = mv^2/r$. Hence $v^2 = gr = 9.8 \times 1.2 =$ **11.76**, so $v =$ **3.43** m/s. M1 for setting $T = 0$, A1 for $mg = mv^2/r$, A1 for the least speed. [3]

SECTION B · Standard

B1 Conserving energy from the bottom to the top, a rise of $2r$, gives $½u^2 = ½v^2 + g(2r)$, so $u^2 = v^2 + 4gr$. The critical case has $v^2 = gr$, so $u^2 = 5gr = 5 \times 9.8 \times 1.2 =$ **58.8** and $u =$ **7.67** m/s. M1 for conserving energy over a rise of $2r$, A1 for $u^2 = v^2 + 4gr$, M1 for using $v^2 = gr$, A1 for the least speed. The result $u^2 = 5gr$ is worth remembering, but the derivation carries the marks and a quoted formula alone will not score them. [4]

B2 It falls a height equal to the radius, so $v^2 = 2gr = 2(9.8)(2) = 39.2$ and $v =$ **6.26** m/s. At the lowest point the radial equation is $R - mg = mv^2/r$, so per unit mass $R = 9.8 + 39.2/2 =$ **29.4** N per kg, three times g. M1 for the energy equation over a drop of r, A1 for the speed, M1 for the radial equation at the lowest point, A1 for the reaction. The reaction is three times the weight at the lowest point, and the extra 2mg is what the circular motion demands. [4]

B3 A rod can push outwards as well as pull inwards, so the constraint never fails and the particle only has to reach the top still moving. That needs $v > 0$ at the top and so $u^2 > 4gr$ at the bottom, here $u > 6.86$ m/s. A string can only pull, so its tension must stay non-negative all the way round, forcing $v^2 \geq gr$ at the top and $u^2 \geq 5gr$ at the bottom. B1 for the rod being able to push, B1 for needing only $v > 0$ at the top, B1 for the string needing $u^2 \geq 5gr$. A bead threaded on a wire or inside a smooth tube behaves like the rod, since the tube can push inwards or outwards. [3]

SECTION C · Stretch

C1 At angle θ the radial equation is $mg\cos\theta - R = mv^2/r$. Energy from the top, having dropped $r(1 - \cos\theta)$: $v^2 = 2gr(1 - \cos\theta)$. The particle leaves when $R = 0$, so $g\cos\theta = v^2/r = 2g(1 - \cos\theta)$. Hence $3\cos\theta = 2$ and **cos** $\theta = 2/3$, that is $\theta = 48.2°$. Substituting back, $v^2 = 2gr(1 - 2/3) = 2gr/3$, so the speed is **$\sqrt{2gr/3}$**. [7]

M1 for the radial equation, A1 for $mg\cos\theta - R = mv^2/r$, M1 for the energy equation, A1 for $v^2 = 2gr(1 - \cos\theta)$, M1 for setting $R = 0$, A1 for $\cos\theta = 2/3$, A1 for the speed.

The result is independent of r and of the mass, so every smooth sphere sheds a particle at the same place. Two equations are needed and each carries marks. The radial equation must resolve the weight along the radius as $mg\cos\theta$, and the energy equation must use the vertical drop $r(1 - \cos\theta)$ rather than an arc length.