



QUESTION WORKBOOK

Numerical methods for differential equations

Further Pure 1 · Year FM

7 questions · 25 marks

NAVY SCREEN EDITION

SECTION A · Warm-up

- A1** Write down the forward and central difference approximations to dy/dx at the point x_n , and say which is more accurate. [2]

- A2** For $dy/dx = x^2 + y$ with $y = 1$ at $x = 0$ and $h = 0.1$, use the forward difference to estimate y at $x = 0.2$. [3]

SECTION B · Standard

- B1** Write down the central difference approximation to the second derivative, and rearrange it to give y_{n+1} . [3]

- B2** Use Simpson's rule with four strips to estimate $\int e^{-x^2} dx$ from 0 to 1, to 4 decimal places. [4]

- B3** Explain why Simpson's rule requires an even number of strips. [2]

- B4** For $y'' = 2x - y$ with $y = 1$ at $x = 0$ and $y = 1.05$ at $x = 0.1$, use the central second-difference approximation with $h = 0.1$ to estimate y at $x = 0.2$ and at $x = 0.3$. [4]

SECTION C · Stretch

- C1** For $dy/dx = x^2 + y$ with $y = 1$ at $x = 0$, use the improved Euler method with $h = 0.1$ to estimate y at $x = 0.2$, giving your answer to five decimal places. The exact solution is $y = 3e^x - x^2 - 2x - 2$. Compare the accuracy of your estimate with the value 1.211 obtained from the forward difference method with the same step length. [7]