inkMaths

ANSWER BOOK

Partial fractions

Algebra and functions · Year 12–13

6 questions · 24 marks

NAVY EDITION

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SECTION A · Warm-up

A1 Set the fraction equal to $A/(x + 1) + B/(x + 2)$ and multiply up, giving $3x + 5 = A(x + 2) + B(x + 1)$. Substituting $x = -1$ kills the B bracket and leaves $2 = A$. Substituting $x = -2$ leaves $-1 = -B$, so $B = 1$. The answer is $2/(x + 1) + 1/(x + 2)$. B1 for the correct form, M1 for the substitutions and A1 for the two constants, so a candidate who writes the form and then stalls still banks one. [3]

A2 At $x = -1$ the factor $(x + 1)$ is zero, so the whole B term vanishes and the equation collapses to a statement about A alone. Each root of the denominator switches off every constant except its own. B1 B1 for the vanishing factor and for the equation in A alone. Comparing coefficients reaches the same values, but it needs simultaneous equations where substitution needs none. [2]

SECTION B · Standard

B1 Set $A/x + B/(x - 1) + C/(x + 2)$ and multiply up: $6x^2 + 5x - 2 = A(x - 1)(x + 2) + Bx(x + 2) + Cx(x - 1)$. Then $x = 0$ gives $-2 = -2A$, so $A = 1$; $x = 1$ gives $9 = 3B$, so $B = 3$; $x = -2$ gives $12 = 6C$, so $C = 2$. The answer is $1/x + 3/(x - 1) + 2/(x + 2)$, and any spare x value, say $x = 2$, checks it: $32 = 4 + 24 + 4$. B1 for the form, M1 for the substitutions, A1 for $A = 1$ and $B = 3$, A1 for $C = 2$. [4]

B2 A squared bracket needs both powers, so try $A/(x + 3) + B/(x + 3)^2$. Multiplying up gives $2x + 7 = A(x + 3) + B$. Substituting $x = -3$ leaves $1 = B$, and comparing the x coefficients gives $A = 2$. The answer is $2/(x + 3) + 1/(x + 3)^2$. B1 for including both powers, M1 for multiplying up, A1 for $B = 1$, A1 for $A = 2$. One fraction per bracket fails here, because a single $A/(x + 3)^2$ cannot produce the $2x$ on the left. The repeated factor demands the pair, and writing it down is the first mark. [4]

B3 The fraction is improper: the top's degree matches the bottom's. Divide first: $x^2 + 4x + 5 = (x^2 + 3x + 2) + (x + 3)$, so the fraction is $1 + (x + 3)/((x + 1)(x + 2))$. Splitting the proper part: $x = -1$ gives $A = 2$, $x = -2$ gives $B = -1$. The answer is $1 + 2/(x + 1) - 1/(x + 2)$. M1 for spotting the improper fraction and dividing, A1 for the quotient and proper remainder, B1 for the partial-fraction form, A1 for $A = 2$, A1 for $B = -1$. Skipping the division and writing two fractions for an improper top is the standard wrong turn. [5]

SECTION C · Stretch

C1 Three constants: $A/(x + 1) + B/(x + 1)^2 + C/(x + 2)$. Multiplying up: $3x^2 + 7x + 8 = A(x + 1)(x + 2) + B(x + 2) + C(x + 1)^2$. Then $x = -1$ gives $4 = B$; $x = -2$ gives $6 = C$; comparing x^2 coefficients, $A + C = 3$, so $A = -3$. The answer is $-3/(x + 1) + 4/(x + 1)^2 + 6/(x + 2)$. B1 for the three-term form, M1 for multiplying up, A1 for $B = 4$, A1 for $C = 6$, M1 for comparing x^2 coefficients, A1 for $A = -3$. A negative constant is not a mistake; the check at $x = 0$ confirms it: $8 = -6 + 8 + 6$. [6]

