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ANSWER BOOK

Polar curves

Polar coordinates · Year FM

7 questions · 27 marks

NAVY EDITION

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SECTION A · Warm-up

A1 $x = 4\cos(2\pi/3) = -2$ and $y = 4\sin(2\pi/3) = 2\sqrt{3}$, so the point is $(-2, 2\sqrt{3})$, in the second quadrant as an argument between $\pi/2$ and π requires. B1 B1 for the two coordinates. [2]

A2 $r = \sqrt{2}$. The point sits in the fourth quadrant, so $\theta = -\pi/4$. B1 for r, M1 for arctan with the quadrant checked, A1 for θ. Quoting $+\pi/4$ from arctan without checking the quadrant is the standard slip; the sketch settles it. [3]

SECTION B · Standard

B1 Multiply by r: $r^2 = 6r\sin\theta$, so $x^2 + y^2 = 6y$. Completing the square: $x^2 + (y - 3)^2 = 9$, a circle of radius 3 centred at $(0, 3)$, sitting on the pole. M1 for multiplying by r, A1 for $x^2 + y^2 = 6y$, M1 for completing the square, A1 for the centre and radius. $r = a\sin\theta$ and $r = a\cos\theta$ are always circles through the origin. [4]

B2 $r = 2\sec\theta$ means $r = 2/\cos\theta$, so $r\cos\theta = 2$. Since $x = r\cos\theta$, the curve is the vertical line $x = 2$, two units to the right of the pole. M1 for $r\cos\theta = 2$, A1 for $x = 2$, B1 for describing the vertical line. A polar equation this short can still be a perfectly straight line. [3]

B3 r = 6, 3 and 0 respectively. The curve bulges to $(6, 0)$ on the initial line, passes three units above the pole, and pinches onto the pole itself at the back, a heart on its side traced once as θ makes a full turn. B1 B1 B1 for the three values of r, B1 for the sketch. [4]

B4 Three petals, each of maximum radius 2. The curve reaches the pole when $\cos 3\theta = 0$, first at $3\theta = \pi/2$, so $\theta = \pi/6$. B1 for three petals, B1 for the maximum r, B1 for $\theta = \pi/6$. An odd multiple of θ gives that many petals; an even multiple gives twice as many, since the negative-r halves land in new positions. [3]

SECTION C · Stretch

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C1 Distance from the initial line is $|y|$, and $y = r \sin \theta = 3(1 + \cos \theta) \sin \theta$. [8]
Differentiating with the product rule gives $dy/d\theta = 3[(1 + \cos \theta)\cos \theta - \sin^2\theta]$. Replacing $\sin^2\theta$ by $1 - \cos^2\theta$ gives $3[2\cos^2\theta + \cos \theta - 1] = 3(2 \cos \theta - 1)(\cos \theta + 1)$. The stationary values are $\cos \theta = 1/2$ and $\cos \theta = -1$, that is $\theta = \pi/3$, $\theta = \pi$ and, by symmetry, $\theta = 5\pi/3$. At $\theta = \pi$ the curve is at the pole, so the greatest distance comes at $\theta = \pi/3$, where $r = 3(3/2) = 9/2$ and $y = (9/2)(\sqrt{3}/2) = 9\sqrt{3}/4 \approx 3.90$, and equally at its mirror image $\theta = 5\pi/3$, the same distance below the line. On the stated interval the points are **(9/2,** $\pi/3$) and **(9/2,** $5\pi/3$). M1 for $y = r \sin \theta$, A1 for the product, M1 for differentiating, A1 for $dy/d\theta$, M1 for using $\sin^2\theta = 1 - \cos^2\theta$ and factorising, A1 for $\cos \theta = 1/2$, A1 for $r = 9/2$, A1 for both $\theta = \pi/3$ and $\theta = 5\pi/3$. Maximising r instead gives $\theta = 0$ and the wrong points entirely; the question asks about distance from the line, so $r \sin \theta$ is the function to differentiate.

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