inkMaths

ANSWER BOOK

Polynomials and the factor theorem

Algebra and functions · Year 12–13

6 questions · 22 marks

NAVY EDITION

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SECTION A · Warm-up

A1 $f(2) = 8 - 16 + 2 + 6 = 0$, so by the factor theorem $(x - 2)$ is a factor. Mind the sign. M1 for substituting $x = 2$, A1 for $f(2) = 0$ with the theorem named. The factor $(x - 2)$ pairs with the root $x = 2$, and substituting -2 instead gives -20, which proves nothing. Name the theorem in the conclusion; a bare zero is only half an answer to a show-that. [2]

A2 Divide the leading terms in turn. First $x^3 \div x = x^2$; multiply back and subtract, leaving $-2x^2 + x$. Then $-2x^2 \div x = -2x$, leaving $-3x + 6$, and finally -3 exactly. The quotient is $x^2 - 2x - 3$ with remainder zero, as the factor theorem promised. M1 for the first division step, dM1 for completing the division, A1 for the quotient. A non-zero remainder here means an arithmetic slip somewhere above, so the zero is a free check on the division. [3]

SECTION B · Standard

B1 The quotient factorises as $x^2 - 2x - 3 = (x - 3)(x + 1)$, so $f(x) = (x - 2)(x - 3)(x + 1)$ and the roots are 2, 3 and -1. M1 for factorising the quotient, A1 for the three brackets, B1 for the roots. Completely means down to linear brackets, so stopping at $(x - 2)(x^2 - 2x - 3)$ loses the final mark. The roots are asked for separately, and they are the values that kill each bracket, not the numbers inside them. [3]

B2 Write the dividend with every power present, as $2x^3 + 3x^2 + 0x - 5$, or the columns drift out of line. Dividing gives quotient $2x^2 - x + 2$ and remainder -9. M1 for the division with the missing term written in, A1 for the quotient, A1 for the remainder. Substitution confirms it, since $f(-2) = -16 + 12 - 5 = -9$, which is the remainder theorem doing the division without dividing. That check takes one line and catches the missing-term error before it costs three marks. [3]

B3 The root that kills $(2x - 1)$ is $x = \frac{1}{2}$, and $f(\frac{1}{2}) = \frac{1}{4} + 5/4 - \frac{1}{2} - 1 = 0$, so the bracket is a factor. Dividing gives $f(x) = (2x - 1)(x^2 + 3x + 1)$. M1 for using $x = \frac{1}{2}$, A1 for $f(\frac{1}{2}) = 0$, M1 for the division, A1 for the quadratic. For a factor $(ax - b)$ the test value is b/a rather than b, so substituting $x = 1$ here proves nothing. The quadratic that comes out has leading coefficient 1, because the 2 has already been used up in the first bracket. [4]

SECTION C · Stretch

C1 Each factor supplies an equation. $f(2) = 8 + 4a + 2b - 30 = 0$ gives $2a + b = 11$, and $f(-3) = -27 + 9a - 3b - 30 = 0$ gives $3a - b = 19$. Adding the two removes b at once: $5a = 30$, so $a = 6$ and then $b = -1$. With $f(x) = x^3 + 6x^2 - x - 30$, the two known brackets multiply to $x^2 + x - 6$, and the constant -30 divided by -6 fixes the third bracket as $(x + 5)$. So $f(x) = (x - 2)(x + 3)(x + 5)$. M1 for $f(2) = 0$, A1 for $2a + b = 11$, M1 for $f(-3) = 0$, A1 for $3a - b = 19$, dM1 for solving the pair, A1 for a and b, A1 for the factorisation. Two factors mean two substitutions and two equations, and setting up both is where the opening marks sit. The last bracket can be read off the constant term rather than divided out, which saves a page of long division. [7]