



QUESTION WORKBOOK

Probability generating functions

Further Statistics 1 · Year FM

6 questions · 25 marks

NAVY SCREEN EDITION

SECTION A · Warm-up

A1 Define the probability generating function of a discrete random variable X , and state the value of $G(1)$. [2]

A2 X has $G(t) = (0.6 + 0.4t)^8$. Name the distribution and find $E(X)$. [3]

SECTION B · Standard

B1 X has $G(t) = (0.6 + 0.4t)^8$, with $E(X) = 3.2$. Find $\text{Var}(X)$ using the generating function. [4]

- B2** Derive the probability generating function of $X \sim \text{Po}(\lambda)$, and use it to confirm $E(X) = \lambda$. [5]

- B3** X and Y are independent with $X \sim \text{Po}(2)$ and $Y \sim \text{Po}(7)$. Use generating functions to identify the distribution of $X + Y$. [3]

SECTION C · Stretch

- C1** A geometric distribution has $G(t) = pt/(1 - qt)$ with $q = 1 - p$. Show that its variance is q/p^2 . The number of trials up to and including the third success is the sum of three independent such variables. Write down its probability generating function and deduce its mean and variance. [8]