



QUESTION WORKBOOK

Rates of change and building differential equations

Differentiation · Year 12–13

8 questions · 29 marks

PRINT EDITION

SECTION A · Warm-up

- A1** A quantity V depends on r , and r depends on time t . Write down the chain rule linking dV/dt , dV/dr and dr/dt , and state which of the three a typical question hands you directly. [2]

- A2** A circular oil slick spreads so that its area increases at a constant rate of 10 m^2 per hour. Find the rate at which the radius is increasing when the radius is 4 m , giving your answer to 3 significant figures. [3]

SECTION B · Standard

- B1** A spherical balloon is inflated so that its volume increases at a constant rate of 120 cm^3 per second. Find the rate at which the radius is increasing when the radius is 4 cm , giving your answer to 3 significant figures. [4]

- B2** A cube expands so that its volume increases at a constant rate of 27 cm^3 per second. Find the rate at which the edge length is increasing at the moment when the edge is 3 cm, and the rate at which the total surface area is increasing at that same moment. [4]

- B3** Write down a differential equation modelling a population P whose growth rate is proportional to the population itself. [2]

- B4** A hot drink cools so that the rate of decrease of its temperature T °C is proportional to the amount by which T exceeds the room temperature of 20 °C. Write down a differential equation for T , defining any constant you introduce. [4]

SECTION C · Stretch

- C1** Water is poured into an inverted right circular cone of semi-vertical angle 30° at a constant rate of $8 \text{ cm}^3 \text{ s}^{-1}$. Find the rate at which the depth of the water is increasing when the depth is 6 cm, giving your answer to 3 significant figures.

[6]

- C2** Water flows into a tank at a constant rate of 30 litres per minute and leaks out at a rate proportional to the volume V litres currently in the tank. Write down a differential equation for V , and given that the volume in the tank eventually settles at 600 litres, find the constant of proportionality.

[4]