inkMaths

ANSWER BOOK

Reciprocal and inverse trigonometric functions

Trigonometry · Year 12-13

8 questions · 29 marks

NAVY EDITION

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SECTION A · Warm-up

A1 sec $x = 1/\cos x$, cosec $x = 1/\sin x$ and $\cot x = 1/\tan x = \cos x/\sin x$. Then sec 60° = $1/(½) = 2$ and cot 45° = $1/1 = 1$. B1 for the three definitions, B1 for sec 60°, B1 for cot 45°. Pair each function with its parent by the third letter, so sec goes with cosine and cosec goes with sine. The mismatch is deliberate and it catches out a fair share of candidates every year. [3]

A2 Divide every term by $\cos^2 \theta$, giving $\sin^2 \theta/\cos^2 \theta + 1 = 1/\cos^2 \theta$, which reads $\tan^2 \theta + 1 = \sec^2 \theta$. M1 for the division, A1 for the identity. Papers ask for exactly this one line, and dividing by $\sin^2 \theta$ instead produces the cosec twin, $1 + \cot^2 \theta = \text{cosec}^2\, \theta$. [2]

SECTION B · Standard

B1 $\sec^2 \theta = 1 + \tan^2 \theta = 1 + 25/144 = 169/144$, so $\sec \theta = 13/12$, taking the positive root because θ is acute. Then cos θ is its reciprocal, 12/13. M1 for the identity, A1 for sec θ, A1 for cos θ. No triangle needs drawing and no angle needs finding, since the identity moves straight from one ratio to the other. [3]

B2 $\cot^2 \theta = \text{cosec}^2\, \theta - 1 = 25/16 - 1 = 9/16$, so $\cot \theta = 3/4$, positive because θ is acute. Since cosec θ = 5/4 gives sin θ = 4/5, and $\cos \theta = \cot \theta \times \sin \theta$, we get $\cos \theta = (3/4)(4/5) = 3/5$. M1 for the identity, A1 for cot θ, M1 for linking cos to cot and sin, A1 for 3/5. The 3-4-5 triangle was underneath the whole question, and spotting it early is a legitimate check rather than a method. [4]

B3 sec x = 2 means cos x = ½, so x = 60° or x = 300°. M1 for flipping to cosine, A1 A1 for the two roots. Every reciprocal equation is solved by turning it back into its parent function first, because the cosine graph is the one whose symmetry you can read. Stopping at 60° loses a mark, since cosine is positive in the fourth quadrant too. [3]

B4 arcsin returns values in $[-\pi/2, \pi/2]$ and arccos returns values in $[0, \pi]$. Then arcsin ½ = $\pi/6$, arccos $(-1) = \pi$ and arctan $1 = \pi/4$. B1 for both ranges, B1 B1 B1 for the three values. The restricted ranges are what make these inverses functions at all. Each has to choose a single angle from infinitely many candidates, and these are the agreed windows. [4]

SECTION C · Stretch

C1 Replace $\sec^2 x$ by $1 + \tan^2 x$ so that the equation involves one function only: $2(1 + \tan^2 x) - 5 \tan x = 1$, which tidies to $2 \tan^2 x - 5 \tan x + 1 = 0$. This does not factorise, so use the formula: $\tan x = (5 \pm \sqrt{17})/4$, giving $\tan x = 0.2192$ or $\tan x = 2.2808$. From the first, $x = 12.4°$ and $192.4°$; from the second, $x = 66.3°$ and $246.3°$. M1 for the identity, A1 for the quadratic, M1 for solving it, A1 for both tan values, M1 for adding 180° to each principal value, A1 for all four roots. Tangent has period 180°, not 360°, so each value of $\tan x$ contributes exactly two solutions in the interval. Looking for the second root at $180° - x$ is the sine habit misapplied, and it gives four wrong answers. [6]

C2 Let $A = \arctan(1/2)$ and $B = \arctan(1/3)$, so $\tan A = 1/2$ and $\tan B = 1/3$. Then $\tan(A + B) = (\tan A + \tan B)/(1 - \tan A \tan B) = (1/2 + 1/3)/(1 - 1/6) = (5/6)/(5/6) = 1$. Now A and B are both positive and both less than $\pi/4$, since their tangents are less than 1, so $A + B$ lies strictly between 0 and $\pi/2$. The only angle in that interval with tangent 1 is $\pi/4$, so $A + B = \pi/4$. M1 for the compound tangent formula, M1 for substituting, A1 for the value 1, A1 for the range argument and the conclusion. The last mark is the one most often dropped. $\tan(A + B) = 1$ alone only narrows the answer to $\pi/4$ plus any multiple of π, and the bounding argument is what pins it down. [4]