inkMaths

ANSWER BOOK

Reduction formulae

Further Pure 2 · Year FM

7 questions · 34 marks

NAVY EDITION

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SECTION A · Warm-up

A1 Take $u = x$ and $dv = e^x\,dx$, so the integral is $[xe^x] - \int e^x\,dx = [xe^x - e^x]$ from 0 to 1 = $(e - e) - (0 - 1) = 1$. M1 for parts with $u = x$, A1 for the integrated expression, A1 for 1. Differentiating the power rather than the exponential is the choice that makes progress, since it lowers the index by one. That is exactly the move a reduction formula automates. [3]

A2 $I_4 = (3/4)I_2 = (3/4)(1/2)(\pi/2) = 3\pi/16$, about 0.5890. $I_5 = (4/5)I_3 = (4/5)(2/3)(1) = 8/15$, about 0.5333. M1 for applying the formula twice to reach I_4, A1 for $3\pi/16$, M1 for the odd chain, A1 for $8/15$. Even n runs down to I_0 and keeps a π; odd n runs down to I_1 and does not. [4]

SECTION B · Standard

B1 Write the integrand as $\sin^{n-1}x \times \sin x$ and integrate by parts, differentiating the first factor. The boundary term is $[-\sin^{n-1}x \cos x]$, which vanishes at both limits, since $\cos(\pi/2) = 0$ and $\sin 0 = 0$. What is left is $(n - 1)\int \sin^{n-2}x \cos^2x\,dx$. Replacing $\cos^2x$ by $1 - \sin^2x$ splits it into $(n - 1)I_{n-2} - (n - 1)I_n$, so $I_n + (n - 1)I_n = (n - 1)I_{n-2}$, which is the result. M1 for splitting off $\sin x$, M1 for integrating by parts, A1 for the boundary term vanishing, M1 for replacing $\cos^2x$, A1 for the printed result. The I_n reappearing on the right is the point of the method; collect it on the left rather than treating it as a new integral. [5]

B2 Integrating by parts with $u = x^n$ gives $[x^ne^x] - n\int x^{n-1}e^x\,dx$. The boundary term is e at $x = 1$ and 0 at $x = 0$, so $I_n = e - nI_{n-1}$. Starting from $I_0 = e - 1$, the chain runs $I_1 = e - (e - 1) = 1$, $I_2 = e - 2$, and $I_3 = e - 3(e - 2) = 6 - 2e$, about 0.5634. M1 for parts with $u = x^n$, A1 for the boundary term, A1 for the printed formula, M1 for $I_0 = e - 1$, dM1 for running the chain, A1 for $6 - 2e$. [6]

B3 Split off $\tan^2x = \sec^2x - 1$, giving $\int \tan^{n-2}x \sec^2x\,dx - I_{n-2}$. The first integral is $[\tan^{n-1}x/(n - 1)]$, which is $1/(n - 1)$ at $\pi/4$ and 0 at 0. So $I_n = 1/(n - 1) - I_{n-2}$. With $I_0 = \pi/4$ this gives $I_2 = 1 - \pi/4$, and $I_4 = 1/3 - (1 - \pi/4) = \pi/4 - 2/3$, about 0.1187. M1 for $\tan^2x = \sec^2x - 1$, A1 for the split, M1 for integrating the first part, A1 for the printed result, M1 for $I_0 = \pi/4$ and the chain, A1 for $\pi/4 - 2/3$. [6]

B4 The formula only relates one integral to another, so it generates a chain rather than a number. You need one integral evaluated directly to anchor it. The index drops by a fixed step, so follow the chain down. A step of 2 lands on I_0 for even n and I_1 for odd n, and a step of 1 always lands on I_0. B1 for the formula only linking one integral to another, B1 for needing one evaluated directly, B1 for choosing the starting value by the step size. [3]

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SECTION C · Stretch

C1 By parts with $u = x^n$, the boundary term $[-x^n e^{-x}]$ is 0 at the origin and tends to 0 at infinity, since the exponential beats any power. What remains is $n\int x^{n-1}e^{-x}\,dx = nI_{n-1}$. Since $I_0 = 1$, repeated use gives $I_n = n!$, so $I_5 = 120$. M1 for parts with $u = x^n$, M1 for the boundary term at infinity, A1 for it vanishing, A1 for the printed formula, B1 for $I_0 = 1$, M1 for the chain, A1 for 120. [7]

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