InkMaths

QUESTION WORKBOOK

Roots of polynomials

Further algebra and series · Year FM

7 questions · 30 marks

NAVY SCREEN EDITION

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SECTION A · Warm-up

A1 The equation $2z^3 + 6z^2 - 4z + 10 = 0$ has roots α, β, γ. Write down $\alpha + \beta + \gamma$ and $\alpha\beta\gamma$. [2]

A2 The quadratic $z^2 - 6z + 4 = 0$ has roots α and β. Without solving, find $\alpha + \beta$, $\alpha\beta$ and $1/\alpha + 1/\beta$. [3]

SECTION B · Standard

B1 The quadratic $z^2 - 6z + 4 = 0$ has roots α and β. Find $\alpha^2 + \beta^2$, and hence write down a quadratic equation with roots α^2 and β^2. [4]

B2 Verify that $z = 2$ is a root of $z^3 - 4z^2 + z + 6 = 0$, find the other two roots, and check the sum and product against the coefficients. [5]

B3 The equation $z^2 - 6z + 4 = 0$ has roots α, β. Find a quadratic equation with integer coefficients whose roots are $\alpha + 1$ and $\beta + 1$. [4]

B4 A cubic with real coefficients has roots that sum to 0 and multiply to -8, and the sum of its pairwise products is -2. Write down the cubic, taking the leading coefficient as 1. [3]

SECTION C · Stretch

C1 The equation $z^3 - 4z^2 + z + 6 = 0$ has roots α, β and γ. Without solving the equation, find $\alpha^2 + \beta^2 + \gamma^2$ and $\alpha^3 + \beta^3 + \gamma^3$, and hence find a cubic equation with integer coefficients whose roots are α^2, β^2 and γ^2. [9]